Operations

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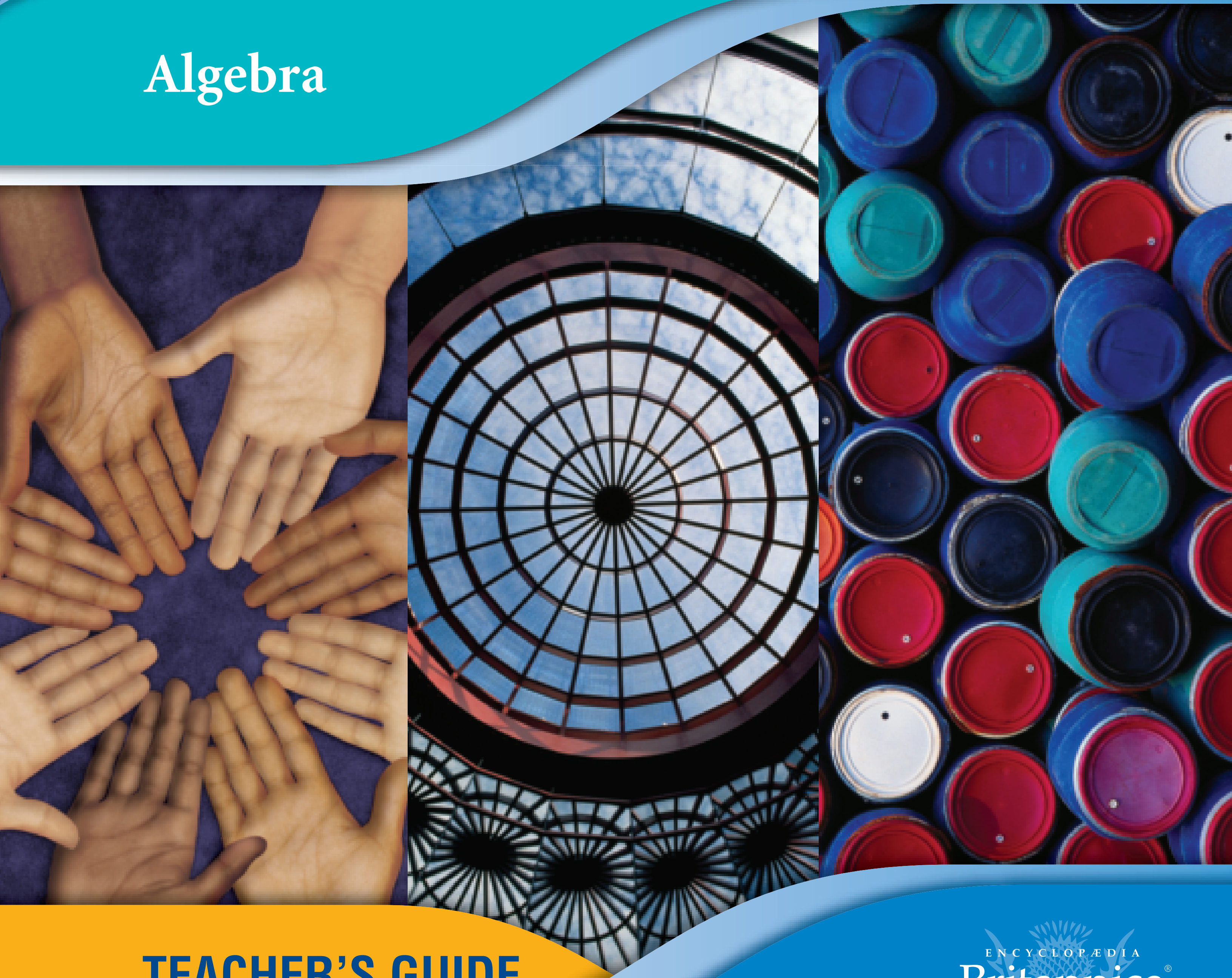
Mathematics in Context

Algebra



TEACHER'S GUIDE

ENCYCLOPÆDIA Britannica®

Mathematics in Context is a comprehensive curriculum for the middle grades. It was developed in 1991 through 1997 in collaboration with the Wisconsin Center for Education Research, School of Education, University of Wisconsin-Madison and the Freudenthal Institute at the University of Utrecht, The Netherlands, with the support of the National Science Foundation Grant No. 9054928.

The revision of the curriculum was carried out in 2003 through 2005, with the support of the National Science Foundation Grant No. ESI 0137414.

National Science Foundation

Opinions expressed are those of the authors and not necessarily those of the Foundation.

International Standard Book Number 978-1-59339-949-8

Printed in the United States of America

1 2 3 4 5 C 13 12 11 10 09

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Development 1991–1997

The initial version of *Operations* was developed by Mieke Abels and Monica Wijers. It was adapted for use in American schools by Gail Burrill, Aaron N. Simon, and Beth R. Cole.

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Contents

Operations and the NCTM Principles and Standards for School Mathematics for Grades 6–8

The process standards of Problem Solving, Reasoning and Proof, Communication, Connections, and Representation are addressed across all *Mathematics in Context* units.

In addition, this unit specifically addresses the following PSSM content standards and expectations:

Number and Operations

In grades 6–8, all students should:

- Develop meaning for integers and represent and compare quantities with them;
- Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers;
- Understand and use the inverse relationships of addition and subtraction, and multiplication and division to simplify computations and solve problems;
- Develop and analyze algorithms for computing with integers and develop fluency in their use.

Algebra

In grades 6–8, all students should:

- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- Relate and compare different forms of representation for a relationship.

Geometry

In grades 6–8, all students should:

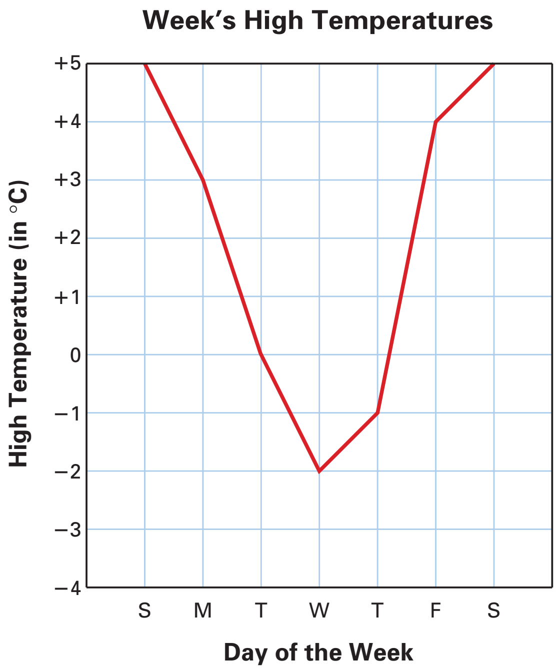
- Describe sizes, positions, and orientations of shapes under informal transformations, such as flips, turns, slides, and scaling.

Math in the Unit

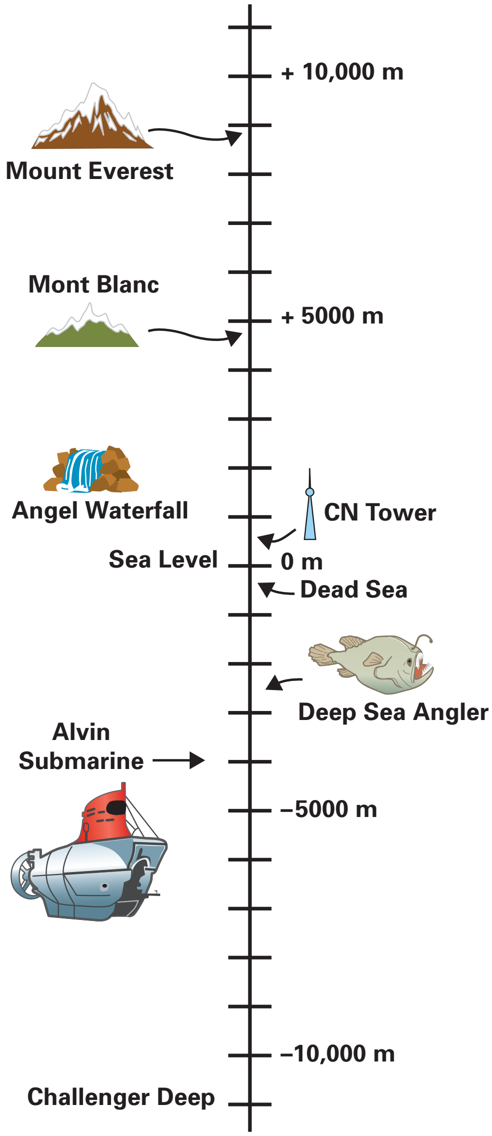
Prior Knowledge

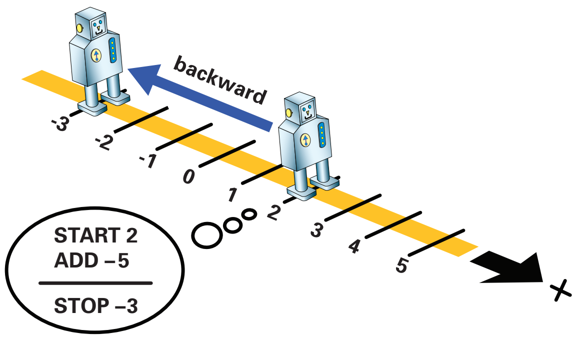
This unit builds on students' informal introduction to the concept of integers, which may have been discussed in earlier grades. We assume students are aware of the existence of negative numbers but have little or no experience with computations with positive and negative numbers.

This unit assumes students can do the following:
- add, subtract, multiply, and divide positive rational numbers;
- recognize, compare, and order positive and negative numbers;
- know the symbols for < (less than) and > (greater than);
- understand the order of operations (as covered in the unit *Expressions and Formulas*);
- use a coordinate system with positive numbers (as introduced in the unit *Expressions and Formulas*);
- use a table to organize data (as covered in *Expressions and Formulas*);
- know the concept of *mean* of a data set (as addressed in *Dealing with Data*);
- know the concept of a number line (see also the *Number Tools* activities for extra practice).



Operations is the third algebra unit and the first one where negative numbers are used by students to model a variety of situations. The concept of positive and negative numbers is revisited in different contexts, such as time zones, sea level, and temperature. Addition and subtraction of integers is informally introduced within these contexts. Students then look at the distance between numbers on a number line.



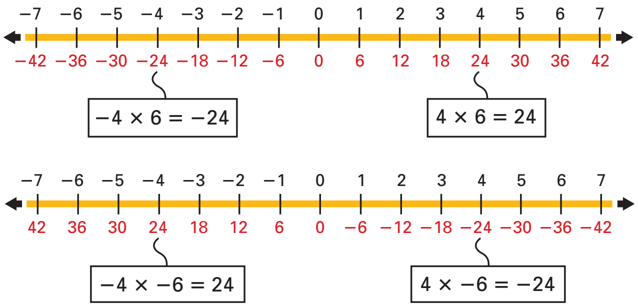


Students explore the number line and order integers. More formal rules for addition and subtraction are introduced, with the help of Ronnie the Robot, who walks along the number line.

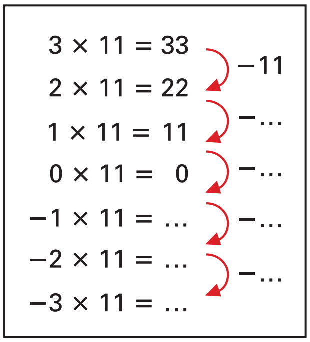
Before starting with multiplication of integers, students have many opportunities to practice adding and subtracting integers, in many different ways. Multiplication is introduced within the context of finding the mean temperature (in degrees Celsius).

In order to explain the rules for multiplication with negative numbers, two different methods are used: a double number line and the “algebraic principle of permanence”.

Double number line

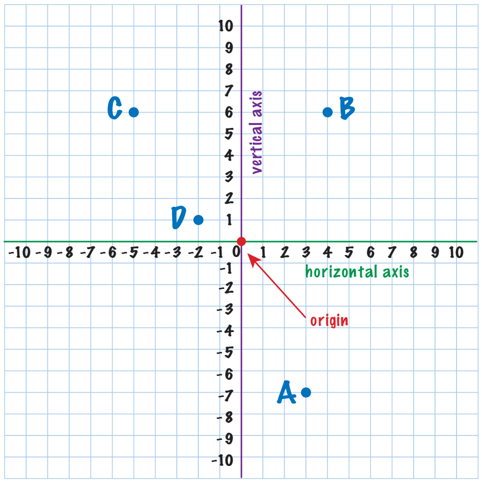


Algebraic principle of permanence



Students practice using the rules for addition, subtraction and multiplication of integers within the context of finding the mean of a large data set of numbers. Formal division of integers is postponed until the unit *Graphing Equations,* where it appears naturally when finding the slope of a straight line, and the unit *Revisiting Numbers.* In *Operations,* students anticipate the division of negative numbers while working on *multiplication trees,* another way to practice working with integers.

In the last section, the coordinate system is expanded with negative numbers. Students plot and interpret points on the coordinate plane. Rules for operations with integers are reinforced by performing transformations of geometric shapes on the coordinate system.



When students have finished the unit, they will:

- understand the concept of integers;
  - Students are able to use integers in many different situations.
- compare and order positive and negative numbers;
  - Students understand and use formal symbols.
  - Students order numbers on a number line.
- perform operations with integers;
- understand and use a coordinate system; and
  - Students transform figures in a coordinate system.
- calculate the arithmetic mean of a data set by looking at deviations to introduce formal multiplication using integers.

Algebra Strand: An Overview

Mathematical Content

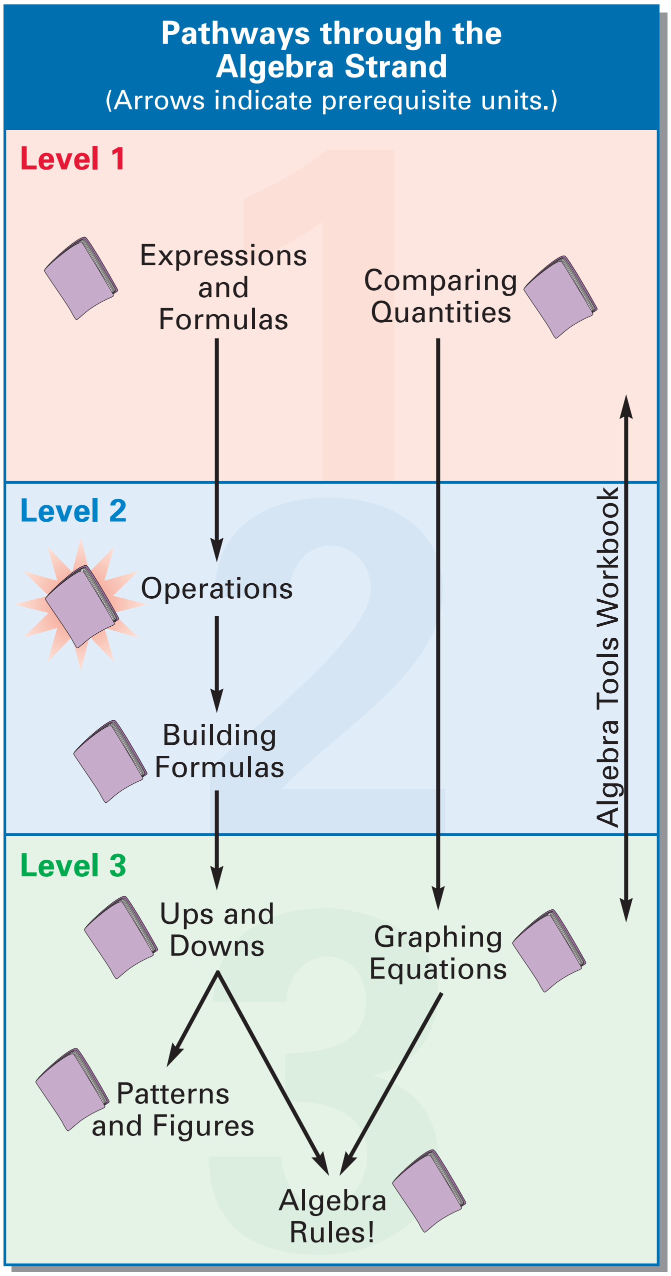
The Algebra strand in *Mathematics in Context* emphasizes algebra as a language used to study relationships among quantities. Students learn to describe these relationships with a variety of representations and to make connections among these representations. The goal is for students to understand the use of algebra as a tool to solve problems that arise in the real world or in the world of mathematics, where symbolic representations can be temporarily freed of meaning to bring a deeper understanding of the problem. Students move from preformal to formal strategies to solve problems, learning to make reasonable choices about which algebraic representation, if any, to use. The goals of the units within the algebra strand are aligned with NCTM's *Principles and Standards for School Mathematics.*

Algebra Tools and Other Resources

The *Algebra Tools* Workbook provides materials for additional practice and further exploration of algebraic concepts that can be used in conjunction with units in the Algebra strand or independently from individual units. The use of a graphing calculator is optional in the student books. The Teacher's Guides provide additional questions if graphing calculators are used.

Organization of the Algebra Strand

The theme of change and relationships encompasses every unit in the Algebra strand. The strand is organized into three substrands: Patterns and Regularities, Restrictions, and Graphing. Note that units within a substrand are also connected to units in other substrands.

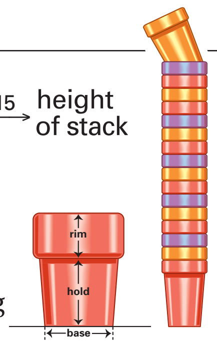


Patterns and Regularities

In the Patterns and Regularities substrand, students explore and represent patterns to develop an understanding of formulas, equations, and expressions. The first unit, *Expressions and Formulas*, uses arrow language and arithmetic trees to represent situations. With these tools, students create and use word formulas that are the precursors to algebraic equations. The problem below shows how students use arrow language to write and solve equations with a single unknown.

The students use an arrow string to find the height of a stack of cups.

number of cups $\xrightarrow{-1}$ ___ $\xrightarrow{\times 3}$ ___ $\xrightarrow{+15}$ height of stack



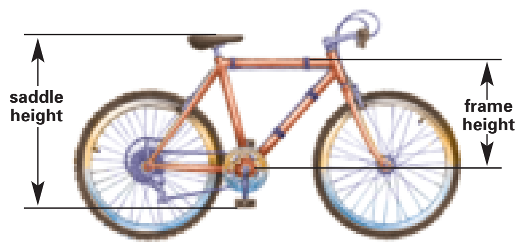
a. How tall is a stack of ten of these cups?

b. Explain what each of the numbers in the arrow string represents.

c. These cups need to be stored in a space 50 cm high. How many of these cups can be placed in a stack? Explain how you found your answer.

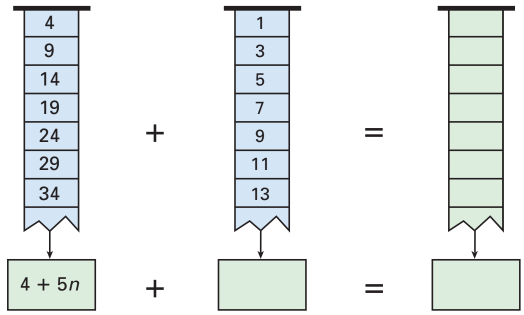
As problems and calculations become more complicated, students adapt arrow language to include multiplication and division. When dealing with all four arithmetic operations, students learn about the order of operations and use another new tool—arithmetic trees—to help them organize their work and prioritize their calculations. Finally, students begin to generalize their calculations for specific problems using word formulas.

saddle height (in cm) = inseam (in cm) × 1.08
frame height (in cm) = inseam (in cm) × 0.66 + 2



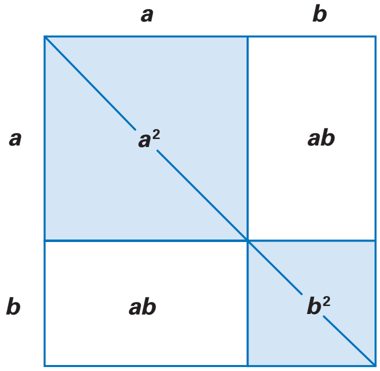
In *Building Formulas*, students explore direct and recursive formulas (formulas in which the current term is used to calculate the next term) to describe patterns. By looking at the repetition of a basic pattern, students are informally introduced to the distributive property. In *Patterns and Figures*, students continue to use and formalize the ideas of direct and recursive formulas and work formally with algebraic expressions, such as $2(n + 1)$.

In a recursive (or NEXT-CURRENT) formula, the next number or term in a sequence is found by performing an operation on the current term according to a formula. For many of the sequences in this unit, the next term is a result of adding or subtracting a fixed number from the current term of the sequence. Operations with linear expressions are connected to “Number Strips,” or arithmetic sequences.



Students learn that they can combine sequences by addition and subtraction. In *Patterns and Figures*, students also encounter or revisit other mathematical topics such as rectangular and triangular numbers. This unit broadens their mathematical experience and makes connections between algebra and geometry.

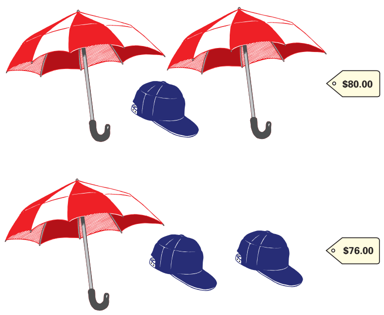
In the unit *Graphing Equations*, linear equations are solved in an informal and preformal way. The last unit, *Algebra Rules!*, integrates and formalizes the content of algebra substrands. In this unit, a variety of methods to solve linear equations is used in a formal way. Connections to other strands are also formalized. For example, area models of algebraic expressions are used to highlight relationships between symbolic representations and the geometry and measurement strands. In *Algebra Rules!*, students also work with quadratic expressions.



The Patterns and Regularities substrand includes a unit that is closely connected to the Number strand, *Operations*. In this unit, students build on their informal understanding of positive and negative numbers and use these numbers in addition, subtraction, and multiplication. Division of negative numbers is addressed in *Revisiting Numbers* and in *Algebra Rules!*

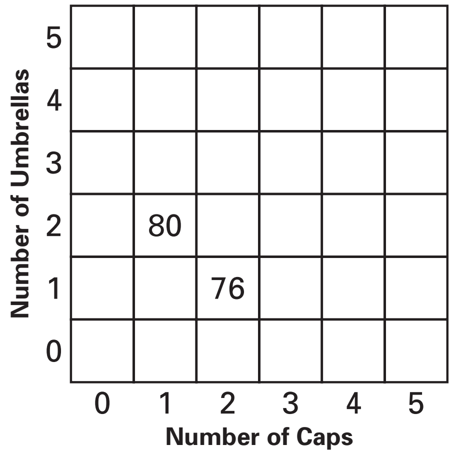
Restrictions

In the Restrictions substrand, the range of possible solutions to the problems is restricted because the mathematical descriptions of the problem contexts require at least two equations. In *Comparing Quantities*, students explore informal methods for solving systems of equations through nonroutine, yet realistic, problem situations such as running a school store, renting canoes, and ordering in a restaurant.



Within such contexts as bartering, students are introduced to the concept of substitution (exchange) and are encouraged to use symbols to represent problem scenarios. Adding and subtracting relationships graphically and multiplying the values of a graph by a number help students develop a sense of operations with expressions.

To solve problems about the combined costs of varying quantities of such items as pencils and erasers, students use charts to identify possible combinations. They also identify and use the number patterns in these charts to solve problems.



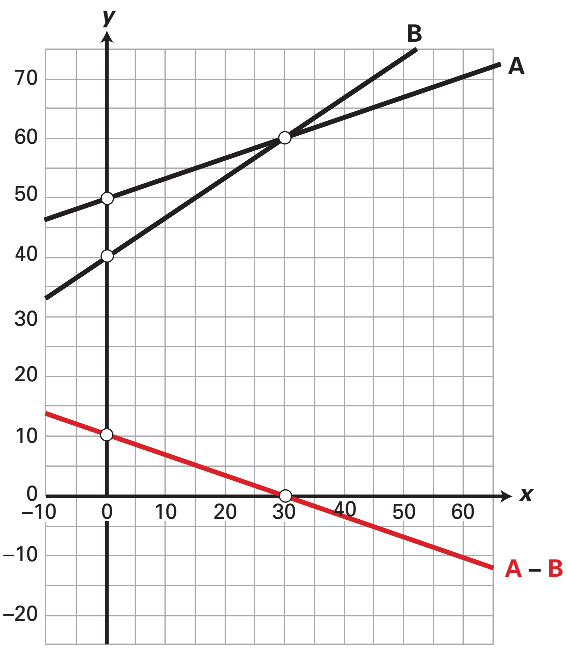
Students' work with problems involving combinations of items is extended as they explore problems about shopping. Given two "picture equations" of different quantities of two items and their combined price, students find the price of a single item. Next, they informally solve problems involving three equations and three variables within the context of a restaurant and the food ordered by people at different tables.

This context also informally introduces matrices. At the end of the unit, students revisit these problem scenarios more formally as they use variables and formal equations to represent and solve problems.

ORDER	TACO	SALAD	DRINK	TOTAL
1	2	4	—	$10
2	1	2	3	$8
3	3	—	3	$9
4	1	2	—	
5	1	—	1	
6	2	2	1	
7	4	2	3	
8				
9				
10				

In *Graphing Equations*, students move from locating points using compass directions and bearings to using graphs and algebraic manipulation to find the point of intersection of two lines.

Students may use graphing calculators to support their work as they move from studying slope to using slope to write equations for lines. Visualizing frogs jumping toward or away from a path helps students develop formal algebraic methods for solving a system of linear equations. In *Algebra Rules!*, the relationship between the point of intersection of two lines (A and B) and the x-intercept of the difference between those two lines (A – B) is explored. Students also find that parallel lines relate to a system of equations that have no solution.

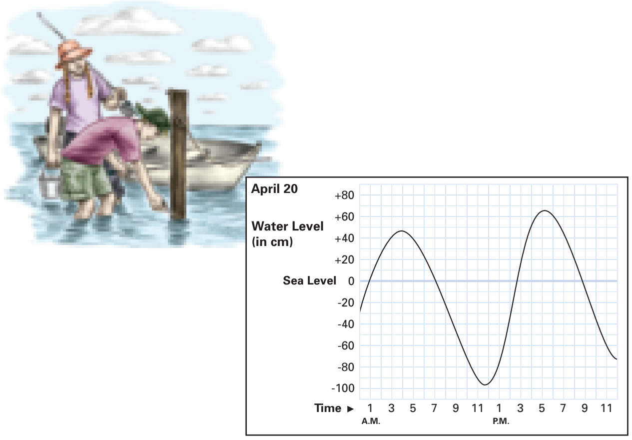


Graphing

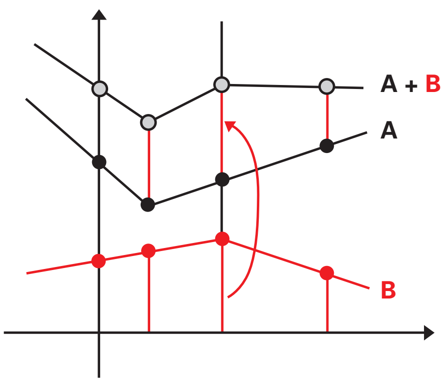
The Graphing substrand, which builds on students' experience with graphs in previous number and statistics units, begins with *Expressions and Formulas* where students relate formulas to graphs and read information from a graph.

Operations, which is in the Patterns and Regularities substrand, is also related to the Graphing substrand since it formally introduces the coordinate system.

In *Ups and Downs*, students use equations and graphs to investigate properties of graphs corresponding to a variety of relationships: linear, quadratic, and exponential growth as well as graphs that are periodic.



In *Graphing Equations*, students explore the equation of a line in slope and y-intercept form. They continuously formalize their knowledge and adopt conventional formal vocabulary and notation, such as origin, quadrant, and x-axis, as well as the ordered pairs notation (x, y). In this unit, students use the slope-intercept form of the equation of a line, $y = mx + b$. Students may use graphing calculators to support their work as they move from studying slope to using slope to write equations for lines. Students should now be able to recognize linearity from a graph, a table, and a formula and know the connections between those representations. In the last unit in the Algebra strand, *Algebra Rules!*, these concepts are formalized and the x-intercept is introduced. Adding and subtracting relationships graphically and multiplying the values of a graph by a number help students develop a sense of operations with expressions.



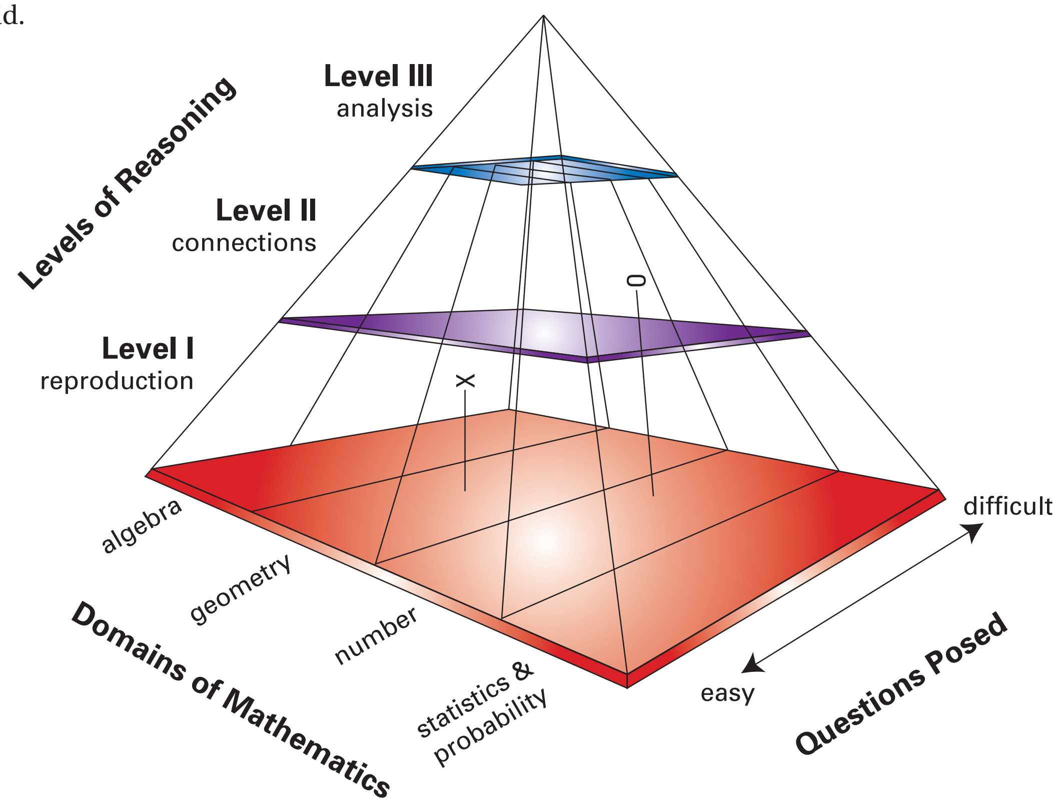
Student Assessment in Mathematics in Context

As recommended by the NCTM *Principles and Standards for School Mathematics* and research on student learning, classroom assessment should be based on evidence drawn from several sources. An assessment plan for a *Mathematics in Context* unit may draw from the following overlapping sources:

- **observation—As students work individually or in groups, watch for evidence of their understanding of the mathematics.**
- **interactive responses—Listen closely to how students respond to your questions and to the responses of other students.**
- **products—Look for clarity and quality of thought in students' solutions to problems completed in class, homework, extensions, projects, quizzes, and tests.**

Assessment Pyramid

When designing a comprehensive assessment program, the assessment tasks used should be distributed across the following three dimensions: mathematics content, levels of reasoning, and difficulty level. The Assessment Pyramid, based on Jan de Lange's theory of assessment, is a model used to suggest how items should be distributed across these three dimensions. Over time, assessment questions should "fill" the pyramid.



Levels of Reasoning

Level I questions typically address:

- recall of facts and definitions and
- use of technical skills, tools, and standard algorithms.

As shown in the pyramid, Level I questions are not necessarily easy. For example, Level I questions may involve complicated computation problems. In general, Level I questions assess basic knowledge and procedures that may have been emphasized during instruction. The format for this type of question is usually short answer, fill-in, or multiple choice. On a quiz or test, Level I questions closely resemble questions that are regularly found in a given unit, substituted with different numbers and/or contexts.

Level II questions require students to:

- integrate information;
- decide which mathematical models or tools to use for a given situation; and
- solve unfamiliar problems in a context, based on the mathematical content of the unit.

Level II questions are typically written to elicit short or extended responses. Students choose their own strategies, use a variety of mathematical models, and explain how they solved a problem.

Level III questions require students to:

- make their own assumptions to solve open-ended problems;
- analyze, interpret, synthesize, reflect; and
- develop one's own strategies or mathematical models.

Level III questions are always open-ended problems. Often, more than one answer is possible and there is a wide variation in reasoning and explanations. There are limitations to the type of Level III problems that students can be reasonably expected to respond to on time-restricted tests.

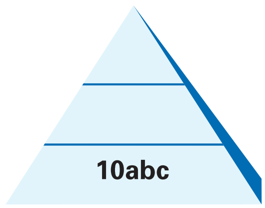
The instructional decisions a teacher makes as he or she progresses through a unit may influence the level of reasoning required to solve problems. If a method of problem solving required to solve a Level III problem is repeatedly emphasized during instruction, the level of reasoning required to solve a Level II or III problem may be reduced to recall knowledge, or Level I reasoning. A student who does not master a specific algorithm during a unit but solves a problem correctly using his or her own invented strategy may demonstrate higher-level reasoning than a student who memorizes and applies an algorithm.

The "volume" represented by each level of the Assessment Pyramid serves as a guideline for the distribution of problems and use of score points over the three reasoning levels.

These assessment design principles are used throughout *Mathematics in Context*. The Goals and Assessment charts that highlight ongoing assessment opportunities—on pages xvi and xvii of each Teacher's Guide—are organized according to levels of reasoning.

In the Lesson Notes section of the Teacher's Guide, ongoing assessment opportunities are also shown in the Assessment Pyramid icon located at the bottom of the Notes column.

Assessment Pyramid

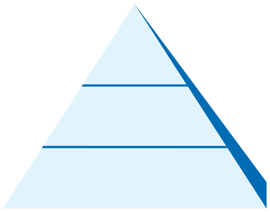


Find a multiplication factor for similar figures.

Determine unknown lengths in given similar figures.

Goals and Assessment

In the *Mathematics in Context* curriculum, unit goals, organized according to levels of reasoning described in the Assessment Pyramid on page xiv, relate to the strand goals and the NCTM *Principles and Standards for School Mathematics.* The *Mathematics in Context* curriculum is designed to help students demonstrate their understanding of mathematics in each of the categories listed below. Ongoing assessment opportunities are also indicated on their respective pages throughout the Teacher's Guide by an Assessment Pyramid icon.

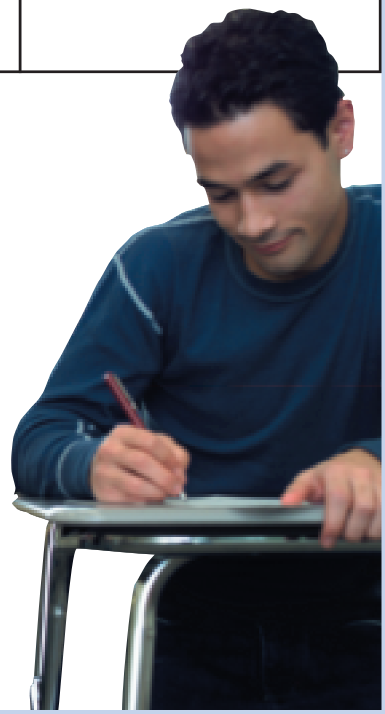


It is important to note that the attainment of goals in one category is not a prerequisite to the attainment of those in another category. In fact, students should progress simultaneously toward several goals in different categories. The Goals and Assessment chart is designed to support preparation of an assessment plan.

	Goal	Ongoing Assessment Opportunities	Unit Assessment Opportunities
Level I: Conceptual and Procedural Knowledge	**1.** Describe patterns using positive and negative numbers.	**Section C** p. 26, #9 **Section D** p. 39, #8ab p. 40, #11	**Quiz 2** #3abcd **Test** #1ab
	2. Compare and order positive and negative numbers.	**Section B** p. 14, #6abcd	**Quiz 1** #2abcde, 3abc **Quiz 2** #3abcd **Test** #2abcd
	3. Perform operations with positive and negative numbers.	**Section B** p. 12, #3 p. 15, #9ab p. 19, #15 **Section C** p. 25, #6ab **Section D** p. 41, #12, 13	**Quiz 1** #1ab, 3abc, 4 **Quiz 2** #2abcd, 3abc, 4 **Test** #3abcdef, 4abcd, 5
	4. Name and plot ordered pairs on a coordinate plane.	**Section E** p. 47, #8ab p. 49, #17b	**Test** #6abcd, 7ab

Level	Goal	Ongoing Assessment Opportunities	Unit Assessment Opportunities
Level II: Reasoning, Communicating, Thinking, and Making Connections	**5.** Recognize and use the property of opposites (canceling out positive and negative numbers).	**Section C** p. 23, #4ab p. 25, #7 p. 30, #17b	**Quiz 1** #1ab, 4 **Test** #3f
	6. Understand the similarity of using integers in algebraic and in geometric contexts.	**Section E** p. 47, #9, 10	**Test** #1ab, 7c
	7. Explore transformations of geometric figures in a coordinate system.	**Section E** p. 47, #9, 10 p. 49, #17a	**Test** #7ab

Level	Goal	Ongoing Assessment Opportunities	Unit Assessment Opportunities
Level III: Modeling, Generalizing, and Non-Routine Problem Solving	**8.** Reason about and predict transformations of geometric figures in a coordinate system.	**Section E** p. 47, #11 p. 51, For Further Reflection	**Test** #7c
	9. Generalize rules for operating with positive and negative numbers.	**Section C** p. 26, #10 p. 35, For Further Reflection **Section D** p. 43, For Further Reflection	**Test** #4e
	10. Use a model or an illustrative context to help solve problems about integers.	**Section A** p. 6, #9ab **Section B** p. 13, #4ab **Section C** p. 30, #18c	



Materials Preparation

The following items are the necessary materials and resources to be used by the teacher and students throughout the unit. For further details, see the Section Overviews and the Materials part of the Hints and Comments section at the top of each teacher page. Note: Some contexts and problems can be enhanced through the use of optional materials. These optional materials are listed in the corresponding Hints and Comments section.

Student Resources

Quantities listed are per student.

- **Letter to the Family**
- **Student Activity Sheets 1–5**

Teacher Resources

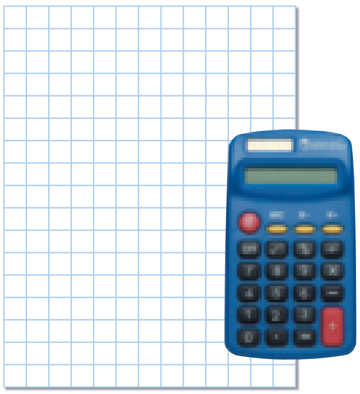
Quantities listed are per group of students.

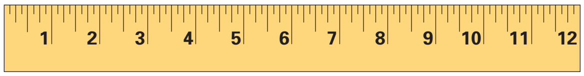
- Colored tape or a rope or long string
- Cards with positive and negative numbers

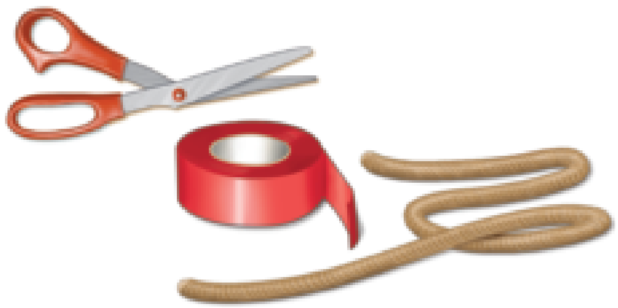
Student Materials

Quantities listed are per pair of students, unless otherwise noted.

- **Calculator**
- **Graph paper (at least three sheets per student)**
- **Index cards (20 per group of two or three students)**
- **Ruler or straightedge (one per student)**
- **Scissors**
- **Tape**





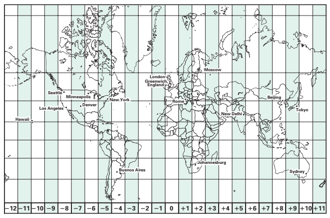


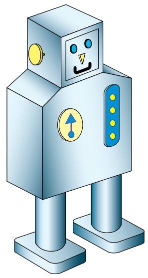
BRITANNICA

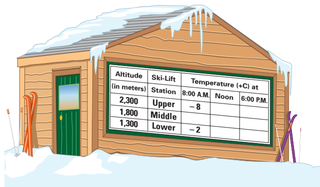
Mathematics in Context

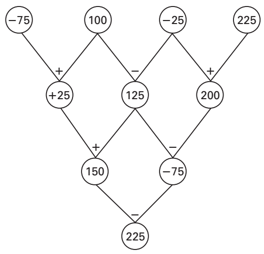
Student Material and Teaching Notes

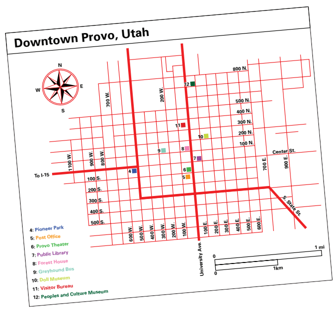
Contents











Dear Student,

Sometimes it is necessary to have numbers that show different directions—or opposites.

Have you ever used positive and negative numbers?

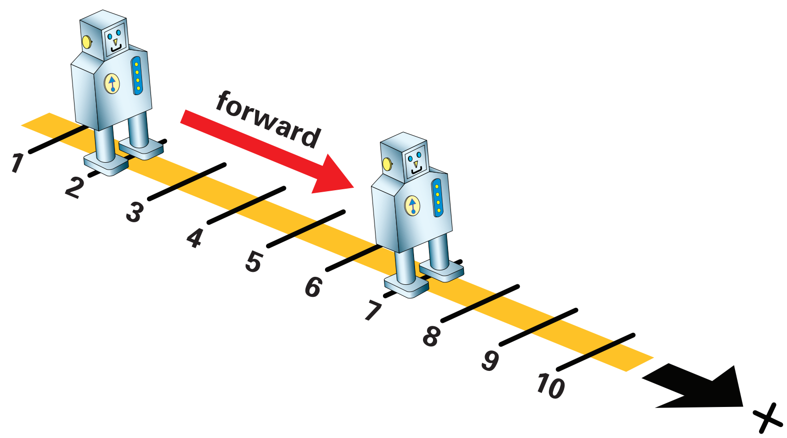
In this unit, you will use a world map to explore time zones and figure out the best times to call people in other parts of the world. You will practice adding, subtracting, and multiplying positive and negative numbers in different contexts. Ronnie the Robot will help you to work with a number line. You will multiply and divide positive and negative numbers to find average temperatures.

In the last section, you will investigate how to move, enlarge, and reduce a shape on graph paper using positive and negative numbers.

We hope you enjoy this unit and learn a lot about operations with positive and negative numbers.

Sincerely,

The Mathematics in Context Development Team



Section Focus

We expect students to be aware of the existence of negative numbers without computing with positive and negative numbers. Positive and negative numbers are revisited within the context of time difference from one location to another and the elevation of places on the earth below and above sea level. The first context involves, for example, Americans watching the live broadcast of a tennis match in Australia. Students use a cylindrical model of the earth with positive and negative numbers to represent the time zone as an introduction to the use of the number line. The number line is reviewed within the context "above and below sea level" where students investigate the difference between highest and lowest points on earth. Informal addition and subtraction of integers starts with counting on the number line.

Pacing and Planning

Day 1: What Time Is It There?		Student pages 1–4
INTRODUCTION	Problems 1 and 2	Discuss three situations that involve a change in time zones.
CLASSWORK	Activity, Page 3 Problem 3	Construct a cylindrical model of the earth to investigate time zones.
HOMEWORK	Problem 4	Use the sun ring on the earth model to solve problems involving time zones.

Day 2: World Time Zones (Continued)		Student pages 4–6 and 52
INTRODUCTION	Problems 5 and 6	Discuss how the positive and negative numbers on the earth model correspond to time zones in the United States.
CLASSWORK	Problems 7–10	Use positive and negative numbers on the cylindrical model of the earth to solve problems involving time change between different time zones.
HOMEWORK	Additional Practice, Section A, page 52	Solve problems involving time change in different time zones.

Day 3: Below and Above Sea Level		Student pages 6–11
INTRODUCTION	Problems 11 and 12	Use positive and negative numbers to describe elevation relative to the sea level.
CLASSWORK	Problems 13–17	Find the difference in elevation between different points on the earth.
HOMEWORK	Check Your Work For Further Reflection	Student self-assessment: Use positive and negative numbers in various situations.

Additional Resources: Additional Practice, Section A, Student Book pages 52–53; *Algebra Tools*, Section B

Materials

Student Resources

Quantities listed are per student.

- Letter to the Family
- **Student Activity Sheet 1**

Teachers Resources

No resources required

Student Materials

Quantities listed are per group of students (unless otherwise indicated).

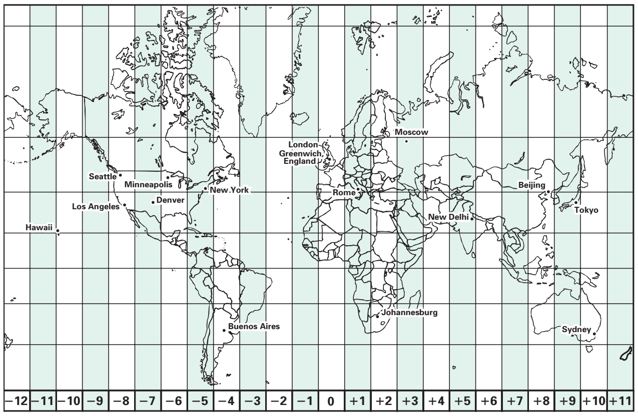
- calculator
- scissors
- tape

* See Hints and Comments for optional materials

Learning Lines

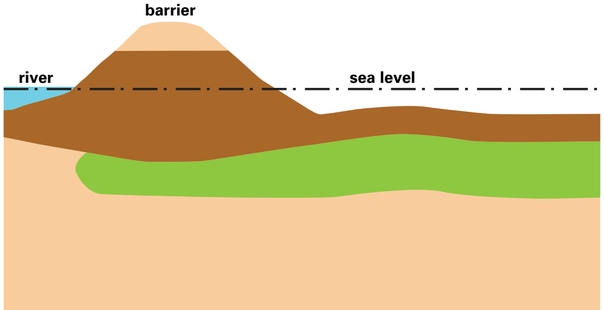
Number Sense

Students review the concept of integers using a variety of contexts. Within the contexts used in this section, they start ordering numbers on the number line. Informal addition and subtraction is introduced by counting on the number line within a context. Zero is neither positive nor negative. Students model a situation involving integers by using a concrete model for time zones on earth.



At the End of This Section: Learning Outcomes

Students are able to model situations using positive and negative numbers.



Notes

You can start this section with a brief discussion of the situations and ask students whether they have had similar experiences.

Be sure to limit the class discussion to one scenario at a time to reduce confusion.

A Positive and Negative

What Time Is It There?

A. Harold and Felicia are big tennis fans. They are watching the Australian Open final being transmitted directly from Australia. Suddenly, their little brother enters the room.

B. It is 7:30 P.M., and Peter knows that tomorrow his cousin Susan is giving her first solo piano recital. He calls her in London to wish her luck.

Reaching All Learners

Act It Out

The situations on this page can be introduced by turning them into skits and role playing the parts.

Extension

You might find a map with the exact time zones on it and use this map as a reference during this section.

Hints and Comments

Overview

Students read about situations that deal with time differences and think of explanations for these situations. There are no problems on this page for students to solve.

About the Mathematics

This section uses the context of time zones and below or above sea level to review the concept of positive and negative numbers and informally introduce methods for adding and subtracting. The number line (with negative and positive numbers), as developed in the number units and in the *Number Tools* resource, can be used to visualize the idea of time zones and what happens when you go from one time zone to another. Jumps on the number line then correspond to jumps from one time zone to another.

Did You Know?

In the United States, there are four zones in the contiguous 48 states and, in addition, the ones in Alaska and Hawaii. In Western Europe, there are only two; all countries except the United Kingdom, Ireland, and Portugal are in the same time zone. So, when you are in Western Europe, you won't experience time differences greater than one hour when you call somebody in another Western European country.

A Positive and Negative

Notes

You may want to use a flashlight and a globe to demonstrate the different times around the world.

Discuss in class why the plane trip "takes longer" even though the trip is the same distance either way. The trip from Minneapolis to Seattle takes $1\frac{1}{2}$ hours. The return trip indicates 11:45 A.M. to 5:15 P.M. Ask, *How many hours is the return trip according to her flight information? Why is it longer by four hours?*

A Fair

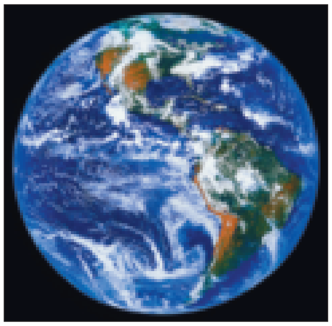
C. Mary is flying from Minneapolis to Seattle. Here is her flight information.

Flight	Date	From/To	Time
NW1607	12/05	Minneapolis to Seattle	11:30 A.M. to 1:00 P.M.

Mary is happy that her trip only takes $1\frac{1}{2}$ hours.

Flight	Date	From/To	Time
NW0008	12/12	Seattle to Minneapolis	11:45 A.M. to 5:15 P.M.

Mary wonders why the trip back from Seattle takes so much longer than the trip to Seattle.



1. These three stories have something in common. Can you explain what it is? This photo of the Earth might help.

2. **Reflect** Have you ever had an experience like those in the three stories? If so, describe it.

Reaching All Learners

Parent Involvement

Encourage students to discuss problem 2 with their families. Have students ask family members whether they have had experiences with time differences and whether they would share and explain those experiences. They can share these examples the following day.

Extension

Develop a pen pal relationship with students in other time zones.

Solutions and Samples

1. Explanations will vary. Some students may realize the situations on the previous page and the one here have to do with time zones. The first two involve communications between different time zones, and the third is about traveling between time zones.

   The picture of Earth shows that one side of the earth is immersed in daylight while the other side is immersed in the dark of night.

2. Answers will vary. Some students may talk about traveling by plane or car across the country or about calling friends or relatives who live far away.

Hints and Comments

Overview

Students answer questions about the three stories on Student Book pages 1 and 2. They put together a model of earth to study time differences.

Planning

You may wish to start with problems 1 and 2 as a whole class activity and have students do the activity in pairs or small groups.

Writing Opportunity

Problem 2 can be assigned as a journal entry. You may want to encourage students to write a story about their experiences.

Did You Know?

Living organisms have rhythms for biological functions such as sleeping, waking, body temperature, heart rate, and tolerance to pain. Scientists believe that an internal device—a "biological clock"—controls these rhythms. A human being's rhythms revolve around a 24-hour cycle. Rhythms slowly reset themselves to coincide with, for example, changes in sleeping habits or in mealtimes. This demonstrates the flexibility of the human body. But the change is slow. If a person moves to a time zone four or more hours away, the body's rhythms will be noticeably out of kilter. The results are lowered efficiency and a sensation of fatigue, a phenomenon known as "jet lag." Within a short time, however, the body's rhythms will change to fit the new cycle.

A Positive and Negative

Notes

Activity

It saves time if you run the time zone maps off on cardstock so that they can be used year after year.

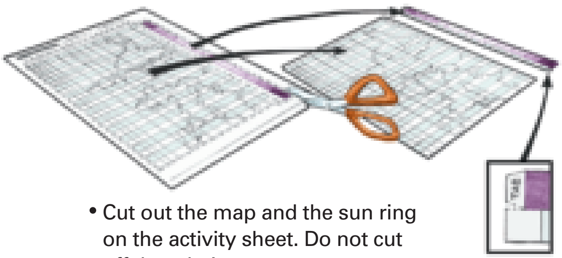
Make sure to specify that the bright sun means noon and the darkened area means midnight.

For more accurate measurements, the cylinder should be tight against the ring. Remind students not to cut off the tabs.

Activity

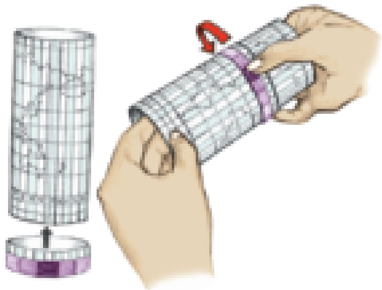
World Time Zones

Student Activity Sheet 1 shows a map that can be folded into a cylinder and a special ring called a **sun ring**. Use **Student Activity Sheet 1** to make the cylinder and the ring. The cylinder is a three-dimensional model of the Earth that can help you answer problem 3.



- Cut out the map and the sun ring on the activity sheet. Do not cut off the tabs!

- Roll the map into a cylinder and tape it closed over the tab. The sections marked +12 and –12 should overlap. Do the same thing for the sun ring.



- Slide the sun ring over the cylinder.

3. a. How can the sun ring on your model explain the photo on the previous page?

b. How can you use the sun ring on your model to explain the first and second stories on page 1?

Reaching All Learners

Accommodation

Some teachers prefer to have students use the time zone map laid flat while the teacher uses the cylinder model.

Extension

The class might discuss that the motion of Earth as it spins around its axis causes the alternation of day and night. This can be modeled in class with a globe. Students might be interested in researching the three motions of Earth: rotating on its axis, revolving around the sun, and moving through the Milky Way as part of our solar system.

Solutions and Samples

3. a. Explanations will vary. Sample response:

When the sun is on one side of earth, it is dark on the other side. So about half of earth is in the light, while the other half is in darkness.

b. Explanations will vary. Sample response:

Australia is on the other side of the earth from the United States. In terms of time zones, the sun ring shows that Australia is about 15 hours ahead of the United States. Peter could also have been on the other side of Earth from Susan in London.

Hints and Comments

Materials

Student Activity Sheet 1 (one per student);
scissors (one pair per group);
tape (one roll per group);
atlas, globe, or encyclopedia map showing time zones, optional (one per class);
cardstock, optional

Overview

Students put together a model of Earth to study time differences.

About the Mathematics

When comparing places in different time zones, you can use words like ahead and behind. These terms, however, may be confusing. It is useful to remember that because earth rotates on its axis from west to east, the sun rises in the east and sets in the west. So places that are farther east are "ahead" of places that are farther west.

Comments About the Solutions

3. Students can find two cities on opposite sides of the cylinder (and the earth) and the corresponding times. They should find that locations that are opposite each other on earth have about a 12-hour time difference.

Students who need more help and/or extra practice.

For some students, especially if they have not had another introduction to positive and negative numbers, you may need to find other examples of situations where positive and negative numbers are used. Think of gains (+) and losses (−) of yardage in football, temperatures below or above zero on a thermometer, and so on. Also, make more problems like problem 3 and help students write statements like:

Going from Johannesburg to Sydney is making a time jump from +2 to +10, which means a difference of eight hours.

Going from Alaska to Chicago is making a time jump from −10 to −6, which means a four-hour difference.

If it is 10:00 o'clock in Salt Lake City, it is 11:00 o'clock in Chicago.

The zero line is in Greenwich, so at one time zone to the right (+1), it is one hour later, and one time zone to the left, it is one hour earlier.

There are 24 hours in a day. Therefore the total number of time zones on the map is 24.

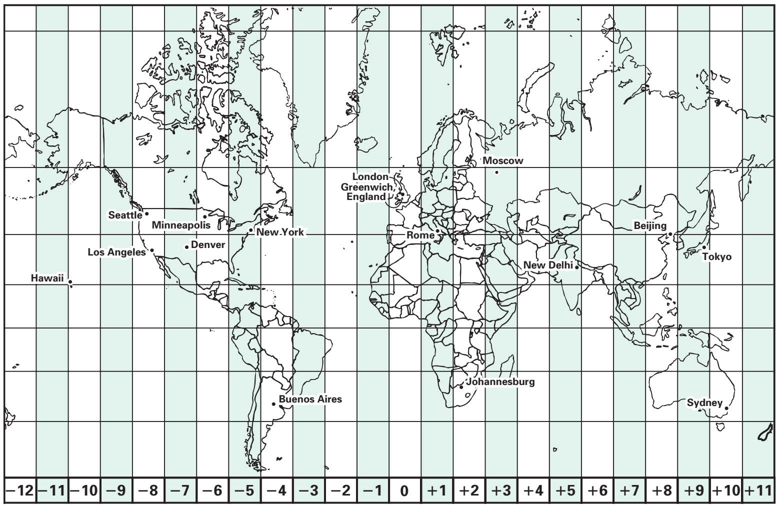
A Positive and Negative

Notes

You may wish to make an overhead of **Student Activity Sheet 1** to use as a visual reference during the class discussion.

4d Students may need to be told that Greenwich is in London, England.

4. a. When it is noon in New York, where is it midnight?
   b. What is the time difference between New York and Los Angeles?
   c. What is the time difference between London and New York?
   d. What does it mean that Greenwich, England, is in the section marked with the number 0?



Note: This map is a simplified version of an actual time zone map, on which the zones often vary to accommodate islands, country borders, and certain geographical features.

This map shows the international **time zones**.

Time is calculated from a zero line in Greenwich. There are 24 zones. When you move from one time zone to the next, you have to change your watch by one hour either backward or ahead, depending on which way you are traveling.

A strip at the bottom of the map shows how the time zones are related to the zero-time zone. For instance, if the time in Greenwich is 9:15 A.M., it is already 10:15 A.M. in Rome (zone marked +1).

You may want to look at an atlas to see the actual time zones.

Reaching All Learners

Extension

Use an atlas to ask more time zone questions while students are looking at actual time zones. For example, you may want to ask, *Starting in the Pacific time zone and moving to the west, how many time zone boundaries do you have to cross before you reach Alaska? Hawaii?* [Alaska is one time zone away from the Pacific time zone. Hawaii is two time zones away.]

Solutions and Samples

4. a. It is midnight roughly anywhere on the vertical line just west of Australia and passing through Malaysia, Indonesia, and China.

b. Three hours. In other words, New York is three hours ahead of Los Angeles.

c. Five hours. That is, London is five hours ahead of New York, so New York is five hours behind London.

d. Students should recognize that time is measured starting from Greenwich, England.

Hints and Comments

Materials

Cylinder model of Earth or **Student Activity Sheet 1** (one per student);
an exact time zone map from an atlas, globe, or encyclopedia, optional (one per class)

Overview

Students explore the time zones in the world.

About the Mathematics

Traveling from one time zone to another corresponds to adding or subtracting a number of hours. When you move to the east, you add hours, while when you move to the west, you subtract.

Note: The map on **Student Activity Sheet 1** and Student Book page 4 is a simplified version of an actual time zone map. In addition, time zones may vary according to local customs, economic factors, and government rulings. Because the lines on the map are approximations, students may find time differences that are not the same as those that exist in reality. However, the focus should remain on using the model to help students understand adding and subtracting positive and negative numbers. Students may want to look at a map to see the actual time zones.

Comments About the Solutions

4. Students should realize from their model that numbers at the bottom edge of the map do not go on forever. In fact, the left end of the map connects with the right end because Earth is round.

Students might also research the International Date Line and the Greenwich meridian, which is often called the prime meridian.

Did You Know?

Greenwich, England has been the home of Greenwich Mean Time (GMT) since 1884. GMT is sometimes called Greenwich Meridian Time because it is measured from the Greenwich Meridian Line at the Royal Observatory in Greenwich.

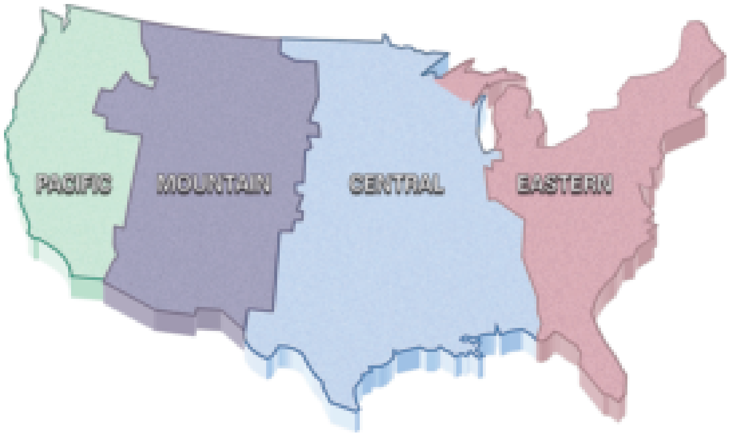
A Positive and Negative

Notes

5b Have a compass rose available for students who have difficulty remembering relative directions.

6b Hawaii is not marked on this map, so you may need to have students look it up in an atlas to complete this problem.

The continental United States has four time zones: Eastern, Central, Mountain, and Pacific.



5. a. Compare the map above to your cylinder map from **Student Activity Sheet 1**. What numbers on the cylinder map correspond to the four time zones shown above?
   b. If you travel from one time zone east to the next time zone on the cylinder map, what happens to the numbers?
   c. If it is 11:30 A.M. Eastern time, what time is it in the Pacific time zone?
   d. When you travel east, should you change the time on your watch one hour backward or ahead?

6. a. On the cylinder map, find the time zone in which you live. What is the number for your time zone? What does this number tell you?
   b. In what time zone is Hawaii? What about Moscow?
   c. Name a city in the time zone marked +2 (which is read as "positive two"). What is the time difference between your time zone and the one in that city? Explain how you found your answer.

Reaching All Learners

Extension Activities

- Have a discussion about how time zones affect television schedules.
- Have students quiz each other on the time differences between different countries.
- Have a discussion about military (Zulu) time and why the military chooses to use Zulu time rather than adapt to different time zones.

Solutions and Samples

5. **a.** Eastern is −5, Central is −6, Mountain is −7, and Pacific is −8.

   b. They go up by one.

   c. 8:30 A.M.

   d. You should move it forward.

6. **a.** Numbers will vary, depending on where the students are located. Explanations will vary. Sample response:

   The number tells me how many hours different we are from time zone 0 in Greenwich, England.

   b. Hawaii is −10. Moscow is +3.

   c. Answers will vary. Athens, Helsinki, Cape Town, Cairo, Juba, Istanbul, Khartoum, Johannesburg, Sofia, Bucharest, Lusaka, Harare, and Gaborone are all in time zone +2.

   Explanations will vary. Sample response:

   We are in time zone −6, so there would be an eight-hour difference between us and Cairo. In time zone +2, it is later than it is in North America.

Hints and Comments

Materials

Cylinder model of earth made from **Student Activity Sheet 1** (one per student);
an exact time zone map from an atlas, globe, or encyclopedia, optional (one per class)

Overview

Students explore the time zones in the United States.

Comments About the Solutions

5. This section can become confusing when students must decide whether a time change is 1 or 2 hours. Students may occasionally get these two mixed up. Help them to think of the operation that they perform to get from one time zone's number to another. For example, a change from 11 to 12 means you should set your watch forward one hour.

Did You Know?

Reference to an exact time zone map shows that actual zones do not always match the rectangular strips in the map on the previous page. For example, eastern Brazil, Uruguay, and Argentina are actually in the same time zone, even though they stretch across more than one rectangle on the map. The same is true in Europe, as pointed out on page 1T.

In many countries, the time changes one hour in the spring and fall. In the spring, clocks are advanced one hour (losing an hour), and in the fall, clocks are set back one hour (gaining the hour back again). Not all countries (not even all the states in the United States) have daylight savings time, and not all countries switch to and from daylight savings time on the same day. As can be imagined, this can cause unforeseen problems in communicating with people in other places. Indiana, for example, does not switch to daylight time in April, so until fall its time matches the eastern zone.

People in the United States (except for military personnel) use a 12-hour time system with A.M. and P.M. In many other parts of the world, people use a time system based on 24 hours. The 0:00 hour is midnight; 12:00 is noon. The hours after noon are continued; thus 1.00 P.M. is 13:00. The 24-hour system makes it easier to calculate time differences, and you do not have to use A.M. or P.M. after the time.

A Positive and Negative

Notes

7 This works much better if students have a fresh copy of **Student Activity Sheet 1** to label where different characters live. Use figures to represent the characters in the story.

9 If students have difficulty answering this problem, discuss why you have to move to the west to find a time two hours earlier.

10 Answers depend on whether or not it is a school day.

Tara, Victor, and José are classmates. The three students live in a city in time zone −5 (which is read as "negative five"). They have relatives who live in countries all over the world. Tara's cousin Keisha lives in a place where they are six hours ahead of where Tara lives.

7. In what time zone does Keisha live? In what countries might Keisha live?

Victor's grandfather lives in time zone +5.

8. What is the time difference between where Keisha lives and where Victor's grandfather lives?

José's uncle lives in a place where it is two hours earlier than where José lives.

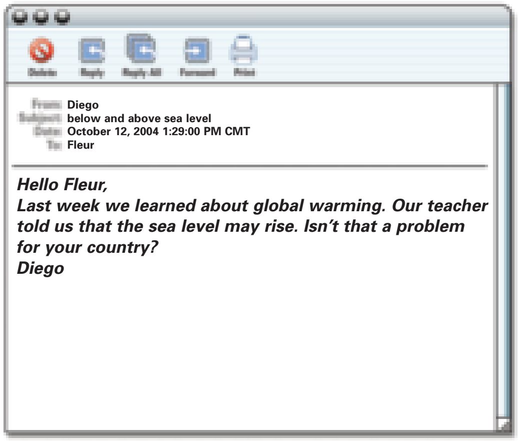
9. **a.** What is the time difference between where Keisha lives and where José's uncle lives?

b. How did you find this difference?

10. At 4:00 P.M. Keisha wants to phone Tara. Is it a good time to call? Why or why not?

Numbers like +2 are called **positive numbers**.
Numbers like −5 are called **negative numbers**.

Below and Above Sea Level



Fleur's class in Nieuwerkerk, The Netherlands, regularly exchanges e-mails with Diego's class in Eagle, Colorado. Diego wrote this e-mail to Fleur:

Reaching All Learners

Vocabulary Building

You may want to encourage students to use the words positive and negative while talking about different time zones.

Advanced Learners

Find where someone would live if his or her time was 8 hours ahead of or behind Tara's time.

Solutions and Samples

7. Keisha lives in time zone +1, where the time is six hours later than in time zone −5. Students may say that Keisha might live in Angola, Nigeria, Italy, Germany, Denmark, or Sweden.

8. There is a four-hour difference (from zone +1 to zone +5). Victor's grandfather lives in a zone where it is four hours later than in Keisha's time zone.

9. a. There is an eight-hour difference (from zone +1 to −7). José's uncle lives where it is eight hours earlier than where Keisha lives.

b. Strategies will vary. Some students may have started at +1 on strip A and counted the number of time zones to −7. There are eight time zones. Other students may have decided that +1 is one hour east of Greenwich and −7 is 7 hours west of Greenwich, then added 7 + 1 to get 8 hours.

10. Answers will vary. Sample response:

When it is 4 P.M. at Keisha's, it will be 10 A.M. at Tara's. If it is a school day, Tara will still be in school, so it might not be a good time to call.

Hints and Comments

Materials

cylinder model of earth or **Student Activity Sheet 1** (one per student)

Overview

Students further explore the time zones in the United States and figure out the time differences between different places on earth. They are also introduced to a new context, below and above sea level.

About the Mathematics

The strip below the map is like a number line. It indicates how many hours to add or subtract from the zero point. Another way to use the strip is to count up or down from one time zone to another. Depending on the direction of the jumps, the corresponding number of hours is to be added or subtracted.

Comments About the Solutions

Actual time differences between countries may vary because political boundaries do not necessarily follow the longitudinal time zones. For example, Buenos Aires is actually in the preceding time zone, one hour ahead of the rectangular strip our map places it in.

Technology

Students may want to research, using encyclopedias or the Internet, how sea level is determined.

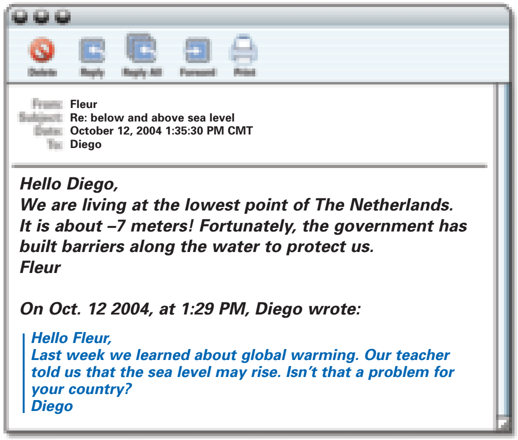
A Positive and Negative

Notes

11 It is important for students to express in words what negative seven or positive six means within a context.

12 Students have great difficulty understanding the term "short notation." You may need to refer back to question 11 and state that −7 m is the short notation for 7 meters below sea level.

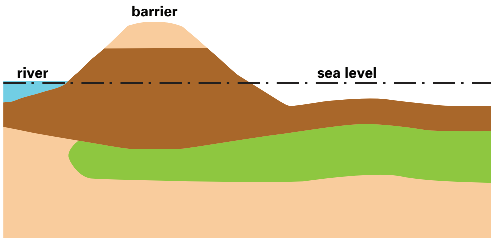
And Fleur answered:


From: Fleur
Subject: Re: below and above sea level
Date: October 12, 2004 1:35:30 PM CMT
To: Diego

Hello Diego,
We are living at the lowest point of The Netherlands. It is about –7 meters! Fortunately, the government has built barriers along the water to protect us.
Fleur

On Oct. 12 2004, at 1:29 PM, Diego wrote:

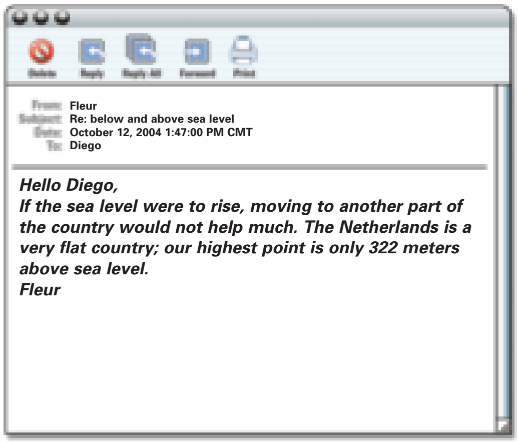
> *Hello Fleur,*
> *Last week we learned about global warming. Our teacher told us that the sea level may rise. Isn't that a problem for your country?*
> *Diego*



11. What does "−7 meters (m)" mean?

12. The height of the barrier Fleur mentioned is about 6 m above sea level. Find a short notation for "6 m above sea level."

In her next e-mail, Fleur wrote:


From: Fleur
Subject: Re: below and above sea level
Date: October 12, 2004 1:47:00 PM CMT
To: Diego

Hello Diego,
If the sea level were to rise, moving to another part of the country would not help much. The Netherlands is a very flat country; our highest point is only 322 meters above sea level.
Fleur

Reaching All Learners

Intervention

Make an overhead of the sea level graphic (or a similar side view sketch) to mark the different points referred to in the emails. Mark these elevations using + and − signs.

Solutions and Samples

11. −7 m means seven meters below sea level (in this context).

12. +6 m (positive 6).

Hints and Comments

Materials

transparency of the sea level graphic, optional

Overview

Students explore positive and negative numbers in the new context, of below and above sea level.

About the Mathematics

Students use numbers like −7 and +6 within the meaning of the context. The imaginary number line is now vertical, in contrast to the first context. In the next section, the number line will be used separate from a context, but students may still refer to the contexts they have seen.

Planning

For some students, it may be necessary to explain how a barrier protects the land below sea level. You may want to discuss how polders were made (that is, new land recovered from the ocean), by building dams and dikes and pumping the water out. Ask students whether they know about other places that are below sea level. Discuss why it is useful to use negative numbers for "below sea level," and you may use actual information from maps showing negative numbers with contour lines. Problems 11 and 12 can be done as a whole class activity.

Did You Know?

Several places in the United States are below sea level as well. For example, nearly 550 square miles of the area of Death Valley National Park is below sea level. Its plants and animals are representative of the Mojave Desert.

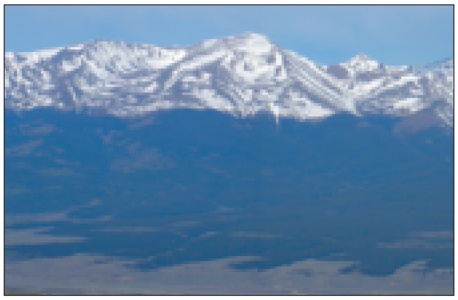
The Salton Sea is currently 228 feet below sea level. Interestingly, the bed of the Salton Sea is only five feet higher than the lowest spot in Death Valley.

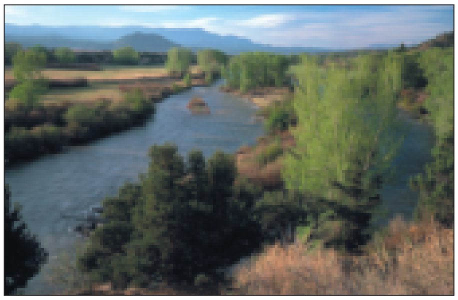
Positive and Negative

Notes

13b This problem requires extra explanation about above and below sea level, and guidance back to the e-mail.

13b Discuss with students why it is necessary to add 7 to 1021. Some students immediately subtract when reading the word "difference."





Diego answered that the highest point in Colorado, Mount Albert, has a height of about 4,400 m above sea level, and the lowest point in Colorado, on the Arkansas River, is still 1,021 m above sea level.

13. **a.** What is the difference in height between the highest point in Colorado and the highest point in The Netherlands?

b. What is the difference between the lowest point in Colorado and the lowest point in The Netherlands?

Now Diego became interested in heights and depths as well. He searched the Internet and came up with the lowest point on earth, the Dead Sea.

14. How can Diego write the depth of the Dead Sea in a shortened way?

Reaching All Learners

Intervention

For students who need more experience with contexts involving positive and negative numbers, see the additional questions on page 8T.

Writing Opportunity

Have students write in their notebooks what negative numbers are, using their own words.

Solutions and Samples

13. a. 4,078 m.

$4,400 - 322 = 4,078$

b. 1,028 m.
From the lowest point in The Netherlands to sea level is 7 m; from sea level to lowest point in Colorado is 1,021 m.

14. -394 m.

Hints and Comments

Materials

calculators, (one per student or group of students).

Overview

Students explore the difference in height between different places on earth.

About the Mathematics

On this page, students practice adding and subtracting integers in an informal way still. The difference in height between two points is later referred to as the distance between two numbers on a number line.

Planning

Students may work on problems 13 and 14 in pairs or small groups. You may want to review measurements expressed in feet and meters here and the conversion between the two. Note that using an estimation rule like "there are about three feet in one meter" helps students to make a mental representation of the actual height.

Intervention

The following are questions for students who need additional practice with contexts involving positive and negative numbers:

1. Leon lives in Helsinki but his relatives live in Chicago. On New Year's Eve, he wants to wish his relatives "Happy New Year" right when the clock strikes twelve midnight. Why do his relatives in Chicago start laughing when they answer the phone?

2. a. Nadia owes her mother five dollars. How can you write this using a + or − sign?

b. Nadia got $3 for her allowance and received $6 for mowing her grandmother's lawn. She also paid her mother the five dollars she owed her. If her account started with zero dollars, does her account now show a + or − sign? Explain.

Answers:

1. If it is midnight in Helsinki (time zone +2 on the map), in Chicago it is eight hours earlier, so it is four o'clock in the afternoon. You do not wish "Happy New Year" at four o'clock.

2. a. Nadia should write an amount owed with a − sign; so Nadia should use −5. Her mother should use a + sign since Nadia owes her the money.

b. Nadia's account should show a + sign. You could use arrow language:

$$0 \xrightarrow{+3} +3 \xrightarrow{+6} +9 \xrightarrow{-5} +4$$

Positive and Negative

Notes

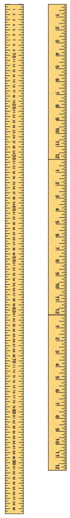
15 Students may have difficulty with the conversions on this page. You could have students practice easier conversions before attempting this problem or set up this problem as a ratio table.

16 Have students approach this problem by estimating. Remind students about the estimation rule that there are about three feet in one meter.

He also found the lowest point in the United States on the Internet. It is in Death Valley, California, and the depth is −282 feet (ft). Diego found that 1 ft = 0.3048 m.

15. Estimate the depth of Death Valley in whole meters. Use a correct notation.

Hint: A meter is about three feet.



Everybody is now interested in record highs and lows. The table below shows a list of heights and depths students found.

Name	Below or Above Sea Level
Florida	lowest: 0 m
Louisiana	lowest: −2.4 m
Alabama	highest: +733 m
Colorado	highest: +4,400 m
Washington, D.C.	lowest: +0.3 m
Nepal	highest: +8,850 m
Challenger Deep	lowest: −11,000 m
Europe	lowest: −28 m

16. What is the lowest point of Washington, D.C., expressed in feet?

17. **a.** Is the lowest point in Florida below, above, or at sea level?

b. Why does the lowest point in Florida not have a plus or a minus sign?

Note: The Challenger Deep is the lowest point in the oceans of the earth. It is situated in the Pacific Ocean, near the Marianas Islands.

If you cut Mount Everest off at sea level and put it on the ocean bottom in the Challenger Deep, there would still be about a mile of water over the top of it!

Reaching All Learners

Visual Learners

Draw a picture or graphic of the relationship between sea level and the low points in each area. You could do this using the side-view graphic on page 7.

Advanced Learners

For problem 15, have students find the exact answer in addition to the estimated answer.

Solutions and Samples

15. Accept any number between −85 and −95 m.

In 1 meter, there are about 3 feet. 270 divided by 3 is 90. So the answer is about −90.

You may also allow your students to use a calculator. $282 \times 0.3048 = 86$. So the answer is −86.

16. About one foot.

17. a. The lowest point of Florida is **at** sea level.

b. The lowest point of Florida is not below or above sea level.

Hints and Comments

Overview

Students find the difference in height between different places on the earth. They use an estimation rule to convert meters to feet.

About the Mathematics

Within the context of heights and depths, the fact that zero is neither positive nor negative is stressed. Large negative numbers are referred to as being "lowest" and large positive numbers as being "highest" as an introduction to the more formal number line, which may also be used vertically or horizontally.

The conversion from feet to meters is brought in here for two reasons. Students will encounter different units in real life, so it is important for them to practice conversions using an estimation rule. The problem is realistic since students, when searching the Internet, are bound to find different units of measurement. When comparing different heights, you need to use the same unit of measurement.

Planning

The *Number Tools* resource, pages 6-9, can be used for additional practice with measurement conversions.

Comments About the Solutions

15. This problem gives you an opportunity to talk about rounding off. Students should see that 85.9536 is an incorrect answer, even if in the question "in whole meters" was left out because the given measurements were not that accurate. And a derived measurement cannot be more accurate than what is given originally.

A Positive and Negative

Notes

Read the summary out loud. Discuss areas in the section where the information was applied.

Be sure to go over the Check Your Work problems with students so that they learn when their answer is correct, even if it does not exactly match the answer given.

3ab Students may have difficulty with the concept of "scale drawings." You may want to discuss what a scale model is or provide examples of scale drawings to make the concept more visual.

A Positive and Negative

Summary

You can use positive and negative numbers in many situations. In this section, you used them for time zones east from the zero line (+) and west of the zero line (−). You also used positive and negative numbers for above sea level (+) and below sea level (−).

Positive numbers are often written with a + in front of the number, but sometimes they are written without it. Either way, they mean the same thing. However, you must write a negative sign for a negative number.

Zero (0) is neither positive nor negative.

Check Your Work

1. Read the story about Mary's trip to Seattle on page 2 again. Did the trip back from Seattle really take longer? Explain your answer.
2. Write down two situations in which you could use positive and negative numbers. Explain how you would use them in the situations you described.

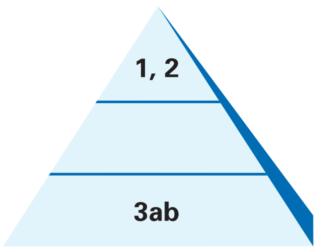
Use the list of heights and depths students found on page 9 to answer the following questions.

"Let's make a scale drawing showing all of the heights and depths from the lowest to the highest point," Erica suggests.

3. a. What is the distance (in meters) between the highest and lowest points on the scale?

   b. Could you make a scale drawing showing the highest and lowest points using a scale of 1:100? Why or why not? Remember that a scale 1:100 means that 1 centimeter (cm) in the drawing equals 100 cm (or 1 meter) in the actual situation.

Assessment Pyramid



Assesses Section A Goals

Reaching All Learners

Parent Involvement

Have students show parents areas in the chapter that relate to the information found in the summary.

Solutions and Samples

Answers to Check Your Work

1. No, the trip back from Seattle did not take longer. Discuss your answer with a classmate. Sample explanation:

   The airline used local times for departure and arrival. There is a two-hour time difference between Seattle and Minneapolis. So 11:45 A.M. in Seattle is 1:45 P.M. in Minneapolis. The flight time was $3\frac{1}{2}$ hours both ways.

2. Some examples of situations where you can use positive and negative numbers:
   - On a thermometer scale in degrees Celsius, below zero is negative (−) and above zero is positive (+).
   - yardage in football. Use + for gains and − for losses of.
   - When you owe somebody money, use −, and when you get money, use +.
   - Use − if the level of water in a lake has dropped below a set level (0) and + if it has risen above the set level.
   - Share your example with the whole class if it was not mentioned here.

3. **a.** The distance between the highest point (8,850 m) and the lowest point (–11,000 m) in the list is 19,850 m.

   b. No, you cannot use scale 1:100. Scale 1:100 means that 1 cm in the drawing equals 100 cm, or 1 m in reality. The length of the scale would be 19,850 cm long (or 198.5 m, which is really long!) Note that in a scale drawing, you can use negative numbers as well.

Hints and Comments

Overview

Reading the Summary, students review situations involving the use of positive and negative numbers. Students use the Check Your Work problems as self-assessment. The answers to these problems are also provided on the Student Book pages 58 and 59.

Planning

After students complete Section A, you may assign as homework appropriate activities from the Additional Practice section, located on Student Book pages 52 and 53.

A Positive and Negative

Notes

Be sure to discuss the Check Your Work section to provide students with a variety of strategies and answer possibilities.

Jassir used positive and negative numbers to show how many meters a trail in the mountains goes up and down. Here is Jassir's table for the Mirror Lake Trail.

Mirror Lake Trail Uphill/Downhill (in m)
+230
−130
+37
−340
+110
−37
+140
−40

4. a. Estimate whether you would end up higher or lower than where you started if you hiked this trail.

Jassir uses the following method to find out exactly how many meters higher or lower the endpoint is.

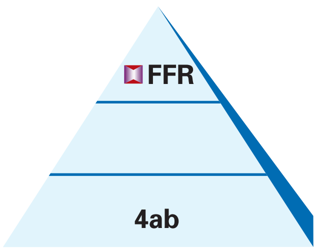
You can cancel +37 uphill and −37 downhill. Some other numbers can be combined too.

b. Find out how much lower the end point is compared to the starting point for Mirror Lake Trail. You may use Jassir's method.

For Further Reflection

Explain why labeling some numbers positive and some negative is helpful. Think of some situations different from those in this section in which you would use positive and negative numbers.

Assessment Pyramid



Assesses Section A Goals

Reaching All Learners

For Further Reflection

Reflective questions are meant to summarize and promote discussion of important concepts.

Solutions and Samples

4. a. You would end up lower than where you started. The total uphill (+) is less than the total downhill (−).

b. You end up 30 m lower than the starting point. Sample strategy:

Cancel out + 37 (uphill) and − 37 (downhill).

+ 230 (uphill) and − 130 (downhill) results in + 100.

+ 100 and + 110 and + 140 results in + 350.

+ 350 and −340 results in + 10.

+ 10 and − 40 results in −30.

Using a drawing to show how much you went uphill and downhill may help.

For Further Reflection

By labeling numbers positive or negative, you can show direction. For example, east or west on the time zone map, up or down in elevation, above or below sea level, and temperature. You need to use positive and negative signs for numbers in situations where zero is used as a marker or starting point (Greenwich, sea level, freezing point for Celsius, etc.)

Hints and Comments

Overview

Students continue working on the Check Your Work problems as self-assessment. The answers to these problems are also provided on Student Book pages 58 and 59.

Section Focus

As the title suggests, the focus of Section B is the number line. The number line is connected to the time zone model from Section A that indicates deviations from Greenwich Mean Time. Number lines are used in a variety of forms—horizontal, vertical and curved—and even a "real" one, as represented by a rope or tape on the classroom floor. Preformal addition and subtraction of integers are introduced with the help of a robot game. Ronnie the Robot walks along the number line while following instructions about adding and subtracting given by the students. For example, with either instruction, ADD −8 or SUBTRACT 8, Ronnie will take 8 steps in a negative direction.

Some statements that are made about a number line used in mathematics:

- A number line can be extended in both directions, infinitely.
- If the number line is horizontal, the positive numbers are to the right of zero and the negative numbers are to the left of zero.
- If the number line is vertical, the positive numbers are above zero and the negative numbers are below zero.
- If you move along the line in the positive direction, the numbers you pass become larger and larger.
- If you move along the line in the negative direction, the numbers become smaller and smaller.
- Decimals and fractions, both positive and negative, can also be indicated on a number line.

If students need more practice working with a number line, additional problems can be found in the Number Tools resource.

Pacing and Planning

Day 4: Ordering the Numbers		**Student pages 12–15**
INTRODUCTION	Problems 1–3	Use a vertical number line for elevation to add and subtract integers.
CLASSWORK	Problems 4–7	Compare and order positive and negative numbers using a number line.
HOMEWORK	Problems 8 and 9	Add and subtract positive and negative numbers.

Day 5: Ronnie the Robot		**Student pages 16–19**
INTRODUCTION	Review homework.	Review homework from Day 4.
CLASSWORK	Problems 10–14 Activity, page 18	Use Ronnie the Robot to model the relationship between adding and subtracting integers.
HOMEWORK	Problem 15	Complete a chain of integer computations.

Day 6: Summary		Student pages 20 and 21
INTRODUCTION	Review homework. Check Your Work	Student self-assessment: Order, compare, and compute with positive and negative numbers.
ASSESSMENT	Quiz 1	Assessment of Sections A and B Goals
HOMEWORK	For Further Reflection	Describe the relationship between adding and subtracting integers.

Additional Resources: Additional Practice, Section B, Student Book page 54; *Number Tools*, Section A; *Algebra Tools*, Section B

Materials

Student Resources

No resources required

Teachers Resources

Quantities listed are per group of students.

- Colored tape or a rope or long string
- Cards with positive and negative numbers

Student Materials

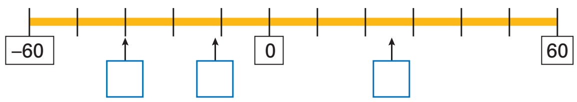
No resources required

* See Hints and Comments for optional materials

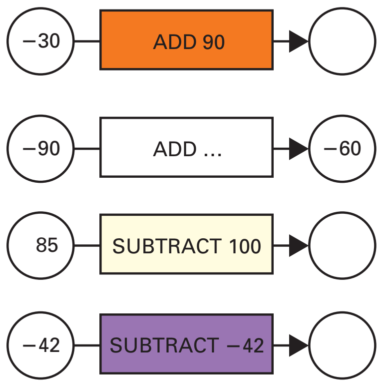
Learning Lines

Number Sense

Students learn to locate and order integers on several types of number lines.



They show on both a context-based and a context -free number line that although five is less than ten, negative five is more than negative ten. Students also use formal notation (<, >, and =) for comparing and ordering integers. Initially, words rather than symbols are used for adding and subtracting integers to avoid confusion about the dual use of the negative sign as an operator and as an indicator of a negative number.



Models

These models of a real number line and the physical experience of being a robot walking forward and backward while carrying out instructions helps students get a better understanding of the operations with integers. The formalized number line is used as a model for ordering, adding, and subtracting integers.

At The End of This Section: Learning Outcomes

Students are able to order integers on a number line and use the formal notations for *less than* (<) and *greater than* (>). They identify the difference between two numbers, for instance 45 and −10 by looking at the distance on the number line from one number to the next. No formal notation for distance is used. Students add and subtract integers in a preformal way. Some students may still prefer to use the robot model.

B Walking Along the Number Line

Notes

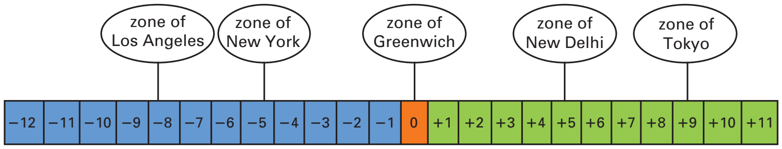
1 Many students will just "count" the number of hours on the number line.

3 Students often try and use subtraction because of the word "difference." Discuss with students what "difference" in this context means. It might help if you use the word "distance" as well. Refer students back to the graphic and show them the space between the two places. Point out to students this "space" is what is being referred to as the "difference."

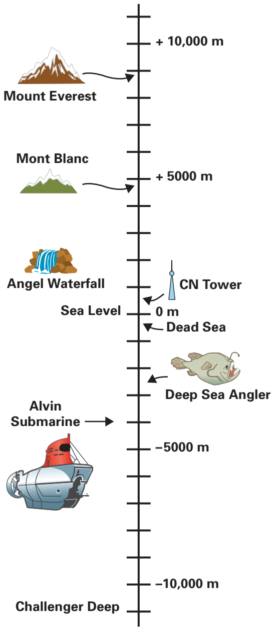
B Walking Along the Number Line

Ordering the Numbers

At the bottom of the time zone map, you saw a strip with 24 numbers, from −12 to +11.



1. What is the time difference between New Delhi and New York? Between New York and Los Angeles?



Number strips or **number lines** can be used for other purposes. For instance, this number line shows heights and depths.

2. a. Mount Everest in Nepal is the highest mountain on earth. From the table on page 9, you can read that its height is 8,850 m. About how many feet is that?

b. The highest mountain in Western Europe is Mont Blanc in Switzerland. About how many meters high is it?

c. At about what depth does the Deep Sea Angler live?

3. What is the difference in height between Mount Everest and Challenger Deep?

Reaching All Learners

Act It Out

Make a large number strip like the one pictured on page 12. Students can walk from one time zone to another and count the number of steps taken.

Intervention

You may want to discuss with students which numbers can be pointed out on a number line. Negative numbers? Fractions? Decimals? Extra practice using a number line can be found in the *Number Tools* resource.

Solutions and Samples

1. New York is in the Eastern time zone (−5).
New Delhi is in the (+5) zone. The time difference between New Delhi and New York is 10 hours.
Los Angeles is in the Pacific time zone (−8).
The difference between New York and Los Angeles is 3 hours.

2. a. Sample student answer:

There are about three feet in one meter. The height of Mount Everest is about

$3 \times 8850 = 26{,}550$ (or 27,000) ft.

b. According to the number line, the height of Mont Blanc is about 4,800 m. Accept any answer between 4,600 m and 4,900 m.

c. The Deep Sea Angler lives between a depth of −2,000 m and −3,000 m.

3. Challenger Deep is about −11,000 m. From Challenger Deep up to the sea level is about 11,000 m. Mount Everest is 8,850 m.

The difference in height between Mount Everest and Challenger Deep is about 19,850 m.

Hints and Comments

Overview

Students review the contexts used in Section A while using number lines.

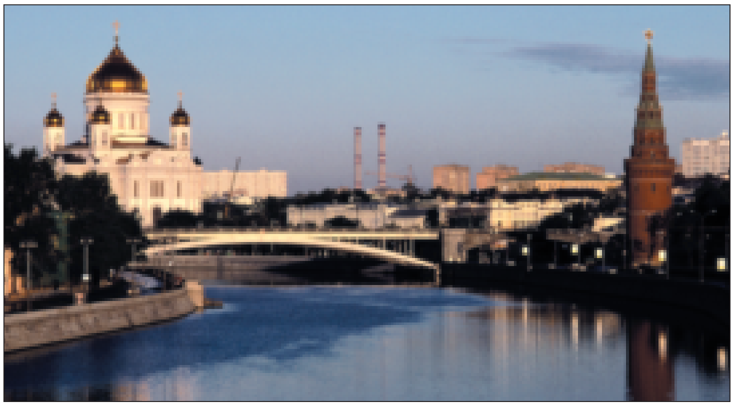
About the Mathematics

In this section, number lines are used horizontally, vertically and even curved, to show that this model can be used in a variety of ways to order, subtract, and add integers.

B Walking Along the Number Line

Notes

4a Encourage students to relate each statement to what they know about ordering and comparing integers.



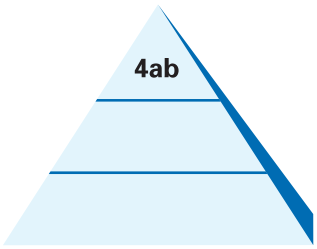
4. **a.** Read the following three statements. Do you agree with them? Explain why or why not.

- In the time zone for New York (−5), it's always earlier than in the zone for Moscow (+3).
- The lowest point in Louisiana (−2.4 m) is lower than the lowest point in Washington, D.C. (+0.3 m).
- If the high temperature was −10 degrees Celsius (−10°C) on Sunday and −2°C on Thursday, it was colder on Sunday than on Thursday.

b. Write three true statements like those above: one comparing times, one comparing altitudes, and one comparing temperatures.

4b You may want to require students to use at least one negative number in their true statements.

Assessment Pyramid



Use a model or an illustrative context to help solve problems about integers.

Reaching All Learners

Hands-On Learning

Have number lines available for students to manipulate differences in time zones.

Intervention

Have a copy of the diagram or picture ready for students to see above and below sea level. Also have a thermometer on hand to physically show students differences in temperatures.

Solutions and Samples

4 a. The statements are true.

- When it is noon in New York, it is already 8 P.M. in Moscow, so it is earlier in New York than Moscow.
- −2.4 is lower than 0.3, and
- −10 is lower (colder) than −2.

b. Answers will vary. Possible student answers:

In Europe it is always *later* than in the US.

Mount Everest is *higher* than Mont Blanc.

The temperature today is 22 degrees Celsius.

That is *warmer* than in January, when the temperature was −5° Celsius.

Students could also write more specific statements. Sample responses:

- In the time zone of L.A. (−8) it's always earlier than in the zone of Tokyo (+9)
- The lowest point of Delaware (+3.4 m) is higher than the lowest point in New York (−5.3)
- If the high temperature on Friday was −9° Celsius and on Saturday it was −6, it was warmer on Friday than Saturday (Note: This is a common misconception that is incorrect.)

Hints and Comments

Overview

Students review the concept of positive and negative numbers within the context of time zones, differences in height and using temperatures expressed in degrees Celsius.

The History of Positive and Negative

In the history of mathematics, negative numbers appeared fairly late in the western world. Fibonacci, also known as Leonardo from Pisa (1170–1250) once wrote, when solving higher order equations, "I can not solve this problem if I do not first assume that this man was in debt!" Remember that long before problems were solved in a generalized way, they were solved within a context. The Chinese, however, were able to use negative and positive numbers much earlier because of the use of their *counting rods*. These rods, originally were made of ivory or bamboo and arranged in columns of lying (Tsungs) and standing (Hengs) rods, representing increasing powers of ten from left to right. Red rods represented positive numbers, and black rods represented the negative ones. Maybe it is a pity we no longer use black and red numbers for the distinction between negative and positive since students get confused by the use of the + and − sign for addition and subtraction as well as positive and negative. That is why we take a lot of time introducing the concept of positive and negative numbers before learning formal operations with them.

The use of the + and − signs for addition and subtraction appeared in print for the first time about the same time Columbus journeyed to America.

Walking Along the Number Line

Notes

Students often have difficulty comparing negative numbers. For example, they initially believe that −6 is greater than −5.

5 Some students may think of the signs as an alligator's mouth, which eats the greater number. This metaphor is usually introduced in earlier grades, but students often remember it.

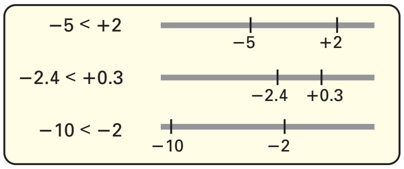
6 You could also compare integers by relating the numbers to money.

Negative = owe

Positive = have

B Walking Along the Number Line

The statements on the previous page can be shortened by statements using numbers.



< means *is less than*

Instead of "earlier," "lower," and "colder," you can use the more general word *less*. So −5 < +2 can be read as −5 is **less than** +2. You can also say: +2 is greater than −5, +0.3 is greater than −2.4, and −2 is greater than −10. The short notation is:

+2 > −5
+0.3 > −2.4
−2 > −10

> means *is greater than*

5. Write words for each of these statements.

a. +7 > −7   **c.** −10 < +9

b. $-6 < -5\frac{3}{4}$ **d.** −1000 > −2000

6. Make true statements using < and >.

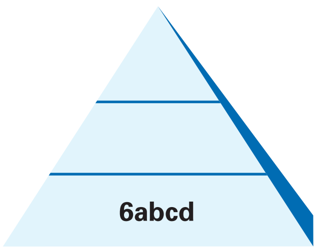
a. 789 ____ 798   **c.** +12 ____ −24

b. −3.7 ____ −4.3   **d.** $\frac{1}{2}$ ____ $\frac{1}{3}$

To help see how the numbers are related, mathematicians use a number line that can be extended in both directions as far as you want! This is shown by the two arrows.

If the number line is horizontal, the *positive* numbers are on the *right* of 0 and the *negative* numbers are on the *left* of 0.

Assessment Pyramid



Compare and order positive and negative numbers.

Reaching All Learners

Intervention

When comparing negative integers, some students find it helpful to know that the number found farther to the right on the number line is always greater.

Hands-On Learning

Provide number strips for students, either on paper or cardstock, and have them tape the strips to their desks. These are also sold in many school supply stores as desk tapes. This will allow students to physically see which is farther to the right.

Solutions and Samples

5. a. +7 is greater than −7.
   b. −6 is less than $-5\frac{3}{4}$.
   c. −10 is less than +9.
   d. −1,000 is greater than −2,000.
6. a. $+789 < +798$
   b. $-3.7 > -4.3$
   c. $+12 > -24$
   d. $\frac{1}{2} > \frac{1}{3}$

Hints and Comments

Overview

Students review the use of the symbols < (less than) and > (greater than) using positive and negative numbers.

About the Mathematics

On this page, the number line using negative as well as positive numbers is introduced. The use of the symbols < and > to order integers is reviewed, and fractions as well as decimals are deliberately used to avoid students thinking that negative numbers are always whole numbers.

Planning

If students have not seen and used the symbols < and > in previous grades, you may want to provide some extra practice. You may want to refer to the context of heights, as presented on page 12, to show the equivalence between numbers becoming larger if you move along the number line in the positive direction. Also, places on earth are higher if you move up in the positive direction along a number line that references height.

Comments About the Solutions

5. Having students write the statements in words (or have them read the statements aloud) strengthens the concept.

B Walking Along the Number Line

Notes

Although some students may just count using the number line, the problems on this page serve as a more formal way of looking at the difference between integers.

8a Have students share their strategies in finding the answers. This will allow students to see easier methods.

Often the positive numbers are written without the +sign, like this.

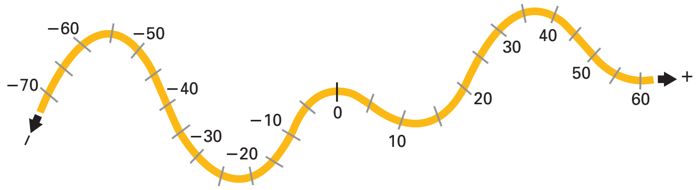
If you move along the line in the positive direction, the numbers that you pass become larger.

If you move in the negative direction, the numbers become smaller. The movement from 4 to -6 is in the negative direction, so $-6 < 4$.

7. **a.** The distance between 4 and -6 is equal to 10. How can you explain that?

b. What is the distance between 14 and -16?

Look at the curved number line.



The difference between 60 and 20 is 40. That is just the distance on the number line!

8. **a.** What is the difference between 45 and -10?

b. What is the difference between -15 and -65?

9. **a.** Give three pairs of numbers, each consisting of a positive and a negative number, with a difference of 100.

b. Give three pairs of negative numbers with a difference of 50.

Reaching All Learners

Accommodation

Use the number line to show distance between numbers. Counting by tens is easiest.

Intervention

Use the curved number line and suggest that students count by fives to see the distance between numbers for problem 8.

Solutions and Samples

7. a. If you move from −6 to 4, you take ten steps on the number line.

b. The distance between 14 and −16 is 30.

8. a. The difference between 45 and −10 is 55.

b. The difference between −15 and −65 is 50.

9. Answers will vary. Possible student answers:

a. −75 and +25

+50 and −50

−10 and +90

b. −75 and −25

−68 and −18

$-52\frac{1}{2}$ and $-2\frac{1}{2}$

Hints and Comments

Overview

Students find the difference between two numbers on a number line.

About the Mathematics

No formal notation for the distance between two numbers on the number line is used. By using the word *distance* instead of *absolute difference,* it should be clear that a positive outcome is expected.

Planning

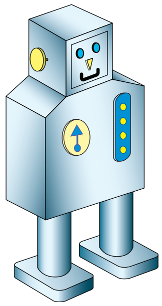
Students may work on problems 7 and 8 in pairs or small groups.

B Walking Along the Number Line

Notes

10 Students may initially have trouble visualizing these problems. You will need to physically show them the movement of Ronnie as the statements are verbalized.

Ronnie the Robot



We can move Ronnie along the number line by giving him an instruction:

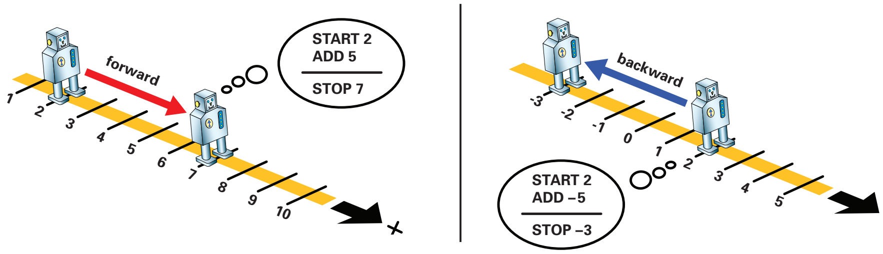
- with one of the two words "ADD" or "SUBTRACT";
- followed by a positive or a negative number.

When the instruction begins with ADD, Ronnie looks in the positive direction.

If the number is positive, he moves forward.

If the number is negative, he moves backward.

Here are two examples:



Suppose Ronnie is standing on the number 2.

The instruction is ADD 5. Ronnie looks in the positive direction and moves forward to the number 7. (See the first picture).

In the second picture, the instruction is ADD −5. Now he moves backward and stops at the number −3.

10. Ronnie starts at the number 2 each time.

a. Where will he stop if the instruction is ADD 18?

b. Where will he stop if the instruction is ADD −18?

11. Now Ronnie starts at the number −5 each time.

a. Where will he stop if the instruction is ADD 5?

b. Where will he stop if the instruction is ADD −5?

Reaching All Learners

Act It Out

Create a life-size number line for your floor and have students take turns playing the role of Ronnie. This allows students to physically see the movement. You may also use a large number line drawn on the board and use a toy robot to show the instructions.

Hands-On Learning

Have students use manipulatives (e.g., plastic human figures) to represent Ronnie. Have them move their figure up and down the number line.

Solutions and Samples

10. a.

START	2
ADD	18
STOP	20

b.

START	2
ADD	−18
STOP	−16

11. a.

START	−5
ADD	5
STOP	0

b.

START	−5
ADD	−5
STOP	−10

Hints and Comments

Materials

Colored tape or a rope or long string and cards with positive and negative numbers, optional

Overview

With the help of Ronnie the Robot who walks the number line, students learn how to add a positive or a negative number.

About the Mathematics

No formal notation for addition and subtraction but the words ADD and SUBTRACT are used in this section to avoid confusion about the positive or negative sign as an operator and the positive or negative sign as an indicator for an integer.

Planning

On page 18, while doing the activity, students will play the role of Ronnie the Robot while walking on a real number line. You may start here by having students show which way Ronnie walks when the instruction ADD is given. Using Ronnie the Robot may help students experience "in the flesh" what the instruction ADD means and get a better understanding of operations with integers.

You may want to work on Problems 10 and 11 as a whole class activity and summarize results with students before you start with subtraction on the next page.

B Walking Along the Number Line

Notes

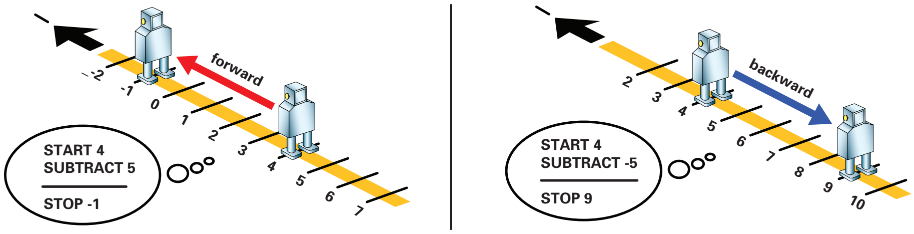
Again, use the life-size number line to demonstrate subtraction.

Students have much more difficulty understanding the subtraction, so you will want to offer extra practice problems here.

Suppose the instruction starts with the word SUBTRACT.

Because of the word SUBTRACT, Ronnie now looks in the negative direction, as you see in the pictures, and:

- if the number is positive, he moves forward.
- if the number is negative, he moves backward.



In the pictures, you see that Ronnie starts at the number 4.

So if the instruction is SUBTRACT 5, he stops at −1.

12 a. If the instruction SUBTRACT 5 is repeated, where does Ronnie stop this time?

b. Where will he stop if the starting point is −4 and the instruction is SUBTRACT −4?

Ronnie is standing at the number −8. You want him to move forward to the number + 8.

13. a. What instruction will you give?

b. But Ronnie wants to move backward! Now, what instruction can you give to have him end at 8?

Now Ronnie is standing at the number −14, and you want to send him to −24.

14. What instruction will you give to have him stop there?

Reaching All Learners

Parent Involvement

Have students make their own Ronnie and take him home. Students will then explain Ronnie's movements to their parents and have their parents solve problems using Ronnie.

Advanced Learners

Give them two number cubes, one red (−), one black (+), and one white operation cube. Have students roll a colored number cube and the operation cube and perform the operation using Ronnie (e.g., A student might roll a red 6 and the subtraction sign. This tells Ronnie to subtract −6).

Solutions and Samples

12. a.

START −1
SUBTRACT 5
STOP −6

b.

START −4
SUBTRACT −4
STOP 0

13. a.

START −8
ADD 16
STOP 8

b.

SUBTRACT −16

14. ADD − 10 or
SUBTRACT 10

Hints and Comments

Materials

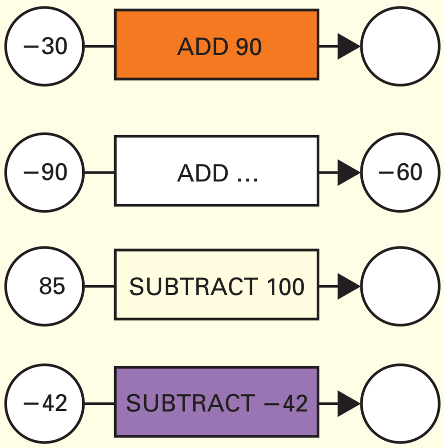
Colored tape or a rope or long string and cards with positive and negative numbers (optional)

Overview

With the help of Ronnie the Robot who walks the number line, students learn how to subtract a positive or a negative number.

Planning

On page 18, while doing the activity, students will play the role of Ronnie the Robot while walking on a real number line. You may start here by having students show which way Ronnie walks when the instruction SUBTRACT is given. Using Ronnie the Robot may help students to experience "in the flesh" what the instruction SUBTRACT means and get a better understanding of operations with integers. If you want to write down instructions for more practice, we suggest you use the notation below, which will be introduced in section C.



Students may work on problems 12–14 in pairs or small groups. Discuss answers in class.

B Walking Along the Number Line

Notes

White boards can be used in place of the number lines on the floor.

Activity

The Robot Game

To play this game, you need a number line on the floor.

You can make this by using colored tape or a rope or long string and cards with positive and negative numbers.

One student plays the role of Ronnie the Robot. Another student chooses from four instructions to move Ronnie.

ADD (+ ...)

ADD (− ...)

SUBTRACT (+ ...)

SUBTRACT (− ...)

Ronnie chooses a starting point. The second student gives an instruction with a number, and Ronnie moves along the line.

The student should give one of each type of instruction in any order. Other students check to see if Ronnie stops on the right spot.

After four moves, the game continues with two other students.

Reaching All Learners

Act It Out

You could also use life-size number lines and take the students outside for this game.

Parent Involvement

Have students play this game with their parents at home.

Hints and Comments

Materials

Colored tape or a rope or long string and cards with positive and negative numbers.

Overview

By representing Ronnie the Robot who walks the number line, students experience how to add and subtract integers while walking a real number line. There are no problems on this page for students to solve.

Planning

You may also wish to draw a chalk number line with positive and negative numbers on the playground to play the Robot Game. Make sure all students have played both roles. If you want students to write the instructions they gave in their notebooks, have them use the words ADD and SUBTRACT.

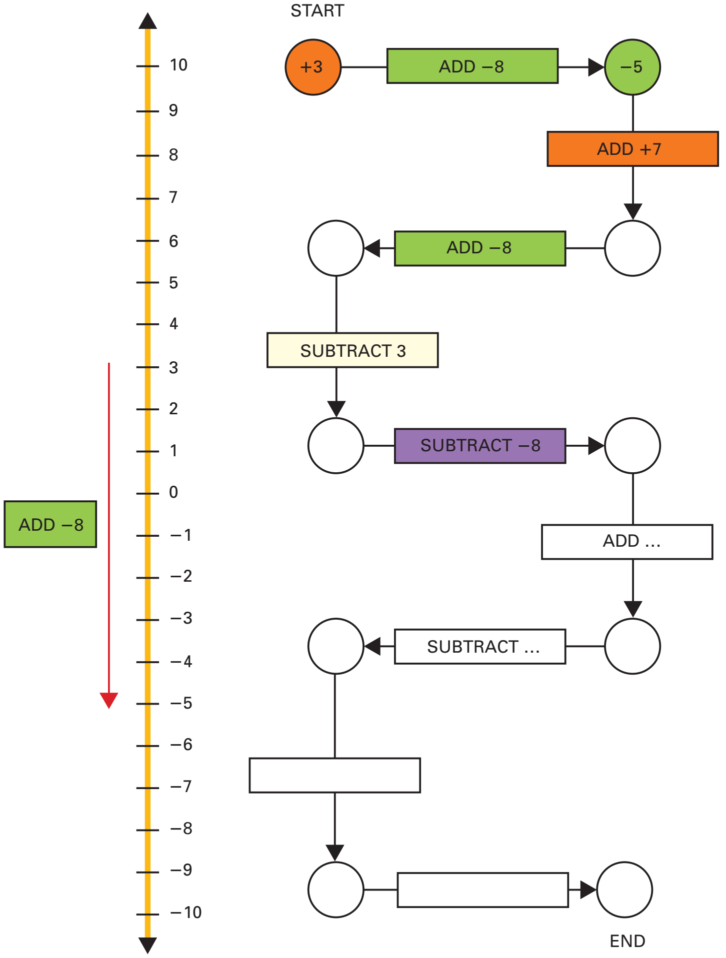
Walking Along the Number Line

Notes

15 Have students come to the overhead and fill in the blanks. Ask them how they found their answer.

15. Complete the following series of instructions. You can use the number line to support your thinking.

At the end of the chain, you can choose your own instructions.



Assessment Pyramid

15

Perform operations with positive and negative numbers.

Reaching All Learners

Accommodation

Allow students to use Ronnie and move him along the number line for each function.

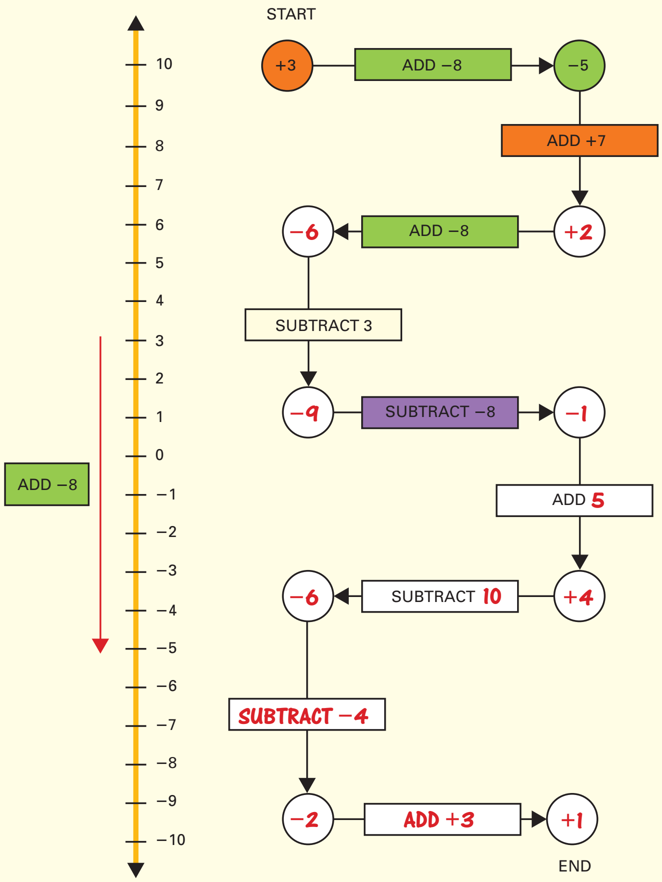
Extension

Have students make their own chain and trade with a partner to solve. Advanced learners can make chains that go above +10 and below −10 so that they can work with higher numbers.

Solutions and Samples

15. At the end of the chain the answers may vary. Discuss different answers.

Sample answer:



Hints and Comments

Overview

Students practice using the instructions ADD and SUBTRACT with integers.

Planning

This problem is an opportunity to practice operations with integers. Encourage students to use the number line drawn on this page if they need to. By the end of the problem, students find their own directions. This will provide you with some information about their progress.

B Walking Along the Number Line

Notes

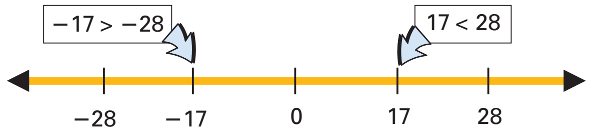
Summary

Have students share information in the Summary with their parents.

B Walking Along the Number Line

Summary

Positive and negative numbers can be ordered on a number line. The farther the number is to the right, the larger the number is. 17 is less than 28, but −17 is greater than −28.



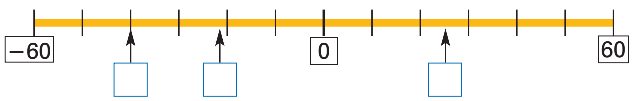
The *difference* between two numbers is the distance between these numbers on the number line; for instance, the difference between 28 and −17 is equal to 45.

Movements along the number line can be indicated through instructions with ADD or SUBTRACT.

ADD 8	⟷	8 steps in the **positive** direction
SUBTRACT 8	⟷	8 steps in the **negative** direction
ADD −8	⟷	8 steps in the **negative** direction
SUBTRACT −8	⟷	8 steps in the **positive** direction

Check Your Work

1. a. Here you see a part of a number line with numbers ranging from −60 to 60. Fill in the blanks.



b. Put one new positive and one new negative number on the number line yourself so that the difference between these numbers is 75.

Reaching All Learners

Intervention

If students are still having difficulty filling in the number line, suggest that they mark it first by tens and then fives.

Solutions and Samples

Answers to Check Your Work

1. a. From left to right, the following numbers should be filled in:

   −40; −22; 25

   b. You can have many different answers. Discuss your answer with a classmate. One example: −45 and 30

Hints and Comments

Overview

Reading the Summary, students review ordering, adding and subtracting positive and negative numbers.

Students use the Check Your Work problems as self-assessment. The answers to these problems are also provided on Student Book pages 59 and 60.

Planning

After students complete Section B, you may assign as homework appropriate activities from the Additional Practice section, located on Student Book page 54.

Notes

Encourage students to discuss their answers and strategies after they have completed the problems in Check Your Work.

2. Make true statements using <, =, or > and write each statement in words.

   a. −24 ____ 14

   b. −2000 ____ 2000

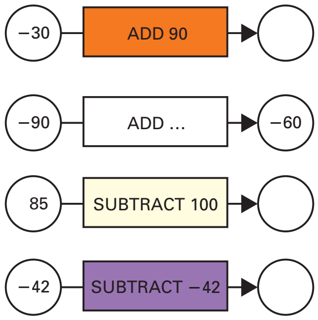
   c. −101 ____ −100

   d. $\frac{1}{4}$ ____ $\frac{1}{5}$

3. a. Ronnie the Robot starts at number 6. The instruction is ADD −9. Where does Ronnie stop?

   b. Write three different instructions for Ronnie. Use ADD as well as SUBTRACT.

4. Complete the following lines.



For Further Reflection

Explain why subtracting −8 is the same as moving 8 steps in the positive direction on a number line.

Assessment Pyramid

FFR

2, 3, 4

Assesses Section B Goals

Reaching All Learners

Parent Involvement

Encourage students to discuss their response to the For Further Reflection problem with family members.

Solutions and Samples

2. a. $-24 < 14$ Negative twenty-four is less than fourteen.

b. $-2000 < 2000$ Negative two thousand is less than two thousand (or 2000).

c. $-101 < -100$ Negative one hundred one is less than negative one hundred.

d. $\frac{1}{4} > \frac{1}{5}$ One fourth is greater than one fifth.

3. a.

START	6
ADD	−9
STOP	−3

b. Discuss your answers with a classmate.
Sample answers:

START	2	START	−2
ADD	−8	SUBTRACT	−4
STOP	−6	STOP	2

4. Use a number line if it it helpful.

−30 (ADD 90) → 60   85 (SUBTRACT 100) → −15

−90 (ADD 30) → −60   −42 (SUBTRACT) → −42) 0

For Further Reflection

Subtracting −8 is the same as Ronnie the Robot turning around toward the negative side of the number line and then moving 8 spaces backwards. This would put Ronnie in the same spot as moving 8 spaces forward in a positive direction.

Hints and Comments

Overview

Students continue working on the Check Your Work problems as self-assessment. The answers to these problems are also provided on Student Book pages 59 and 60.

Section Focus

In this section, students practice addition and subtraction of integers in a more formal way. Through this section, the references to operations change from language-based (informal) to symbolic (formal). The language-based notation from the previous section is reviewed. Then circles are used:

$$\enclose{circle}{-32} + \enclose{circle}{68} = \enclose{circle}{36}$$

$$\enclose{circle}{32} + \enclose{circle}{-68} = \enclose{circle}{-36}$$

Parentheses make it possible to discern negative numbers in a calculation instead of the circles used earlier: $14 - (-3) = 17$ and $14 + (-3) = 11$.

Pacing and Planning

Day 7: Adding and Subtracting		**Student pages 22–25**
INTRODUCTION	Problems 1–3	Determine the floor number of an underground parking garage, given in integers.
CLASSWORK	Problems 4–6	Practice adding and subtracting integers with a number line and arithmetic trees.
HOMEWORK	Problem 7	Practice addition and subtraction with integers using arithmetic trees.

Day 8: Adding and Subtracting (Continued)		**Student pages 26, 27, and 55**
INTRODUCTION	Review homework. Problems 8–11	Review homework from Day 7. Review the context of the elevator ride to subtract integers.
CLASSWORK	Activity, page 27	Practice addition and subtraction with integers with the Integer game.
HOMEWORK	Additional Practice, Section C, Problem 3	Continue adding and subtracting integers using arithmetic trees.

Day 9: Temperatures and Altitudes		**Student pages 28–30**
INTRODUCTION	Problems 12 and 13	Determine the mean high and low temperatures for a week.
CLASSWORK	Problems 14–17	Find the mean of a data set that contains negative and positive numbers.
HOMEWORK	Problem 18	Use the strategy of repeated addition (as multiplication) to find the mean.

Day 10: Higher and Higher		Student pages 30–33
INTRODUCTION	Problems 19 and 20	Discuss the influence of elevation on temperature.
CLASSWORK	Problems 21–24	Determine the temperature at different elevations using the relationship between temperature and elevation.
HOMEWORK	Problems 25 and 26	Solve problems involving elevation and temperature changes.

Day 11: Summary		Student pages 34 and 35
INTRODUCTION	Review homework.	Review homework from Day 10.
CLASSWORK	Check Your Work	Student self-assessment: Compute with positive and negative numbers.
HOMEWORK	For Further Reflection	Describe the relationship between multiplication and addition.

Additional Resources: Additional Practice, Section C, Student Book pages 54 and 55; *Number Tools*; *Algebra Tools*

Materials

Student Resources

Quantities listed are per student.

- **Student Activity Sheet 2**

Teachers Resources

No resources required

Student Materials

Quantities listed are per student, unless otherwise noted.

- Index cards (20 per pair or group of three students)

* See Hints and Comments for optional materials

Learning Lines

Number Sense

Students show their knowledge of the concept of operations with integers on a reflective level by answering questions like: *Is this statement always true? Explain why or why not.*

A repeated addition is written as a multiplication, for example, $(-5) + (-5) + (-5) = 3 \times (-5) = -15$.

To become flexible in carrying out operations with integers, students practice while filling in numbers in addition and subtraction "trees."

Models

The number line is used to strengthen the concept of adding and subtracting integers. The calculation $-32 + 68$ is carried out by splitting 68 into 32 and 36. First add 32 to -32 and arrive at zero on the number line; then add the remaining 36.

At the End of This Section: Learning Outcomes

Students practice adding and subtracting integers using parentheses instead of circles to discern the operation from negative numbers. They develop formal strategies to multiply integers.

C Calculating with Positive and Negative Numbers

Notes

Outside of the United States, elevators often indicate the different floors above and below the ground level with positive and negative numbers.

2a Make sure students understand this context. You may want to discuss 2a as a whole class, to talk through the process. Then have students complete problem 2b on their own.

C Calculating with Positive and Negative Numbers

Adding and Subtracting

Ms. Parker is working in a building that has 40 stories above the ground floor. The building has an underground parking garage with 6 floors. These floors are indicated with negative numbers: −1 through −6.

The ground floor is indicated by 0, and the upper floors have numbers from 1 to 40.

Ms. Parker leaves her car at level −4 and enters the elevator. Then she rides the elevator for 28 floors.

1. At what level does she arrive?

2. The calculation in problem 1 can be shown as an addition.

(−4) —— ADD 28 ——→ (?)

a. Which movement of the elevator corresponds with the following?

(−6) —— ADD 40 ——→ (?)

What is the result of the addition?

b. Which movement of the elevator corresponds with the following?

(24) —— ADD −28 ——→ (?)

What is the result of the addition?

Reaching All Learners

Intervention

Provide students with number line so they can visibly count the floors between −6 and 40. Provide graphical representation of a building that labels various floors.

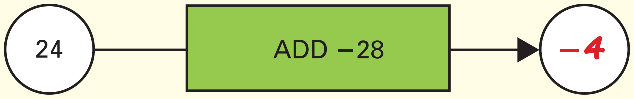
Using circles around the numbers helps students view the "operation" sign separate from the positive/negative sign. In this section, use these circles (or parentheses) initially to distinguish positive and negative signs from the operations. As students become more comfortable with adding and subtracting integers, you may choose to reduce the use of circles.

Solutions and Samples

1. Ms. Parker will arrive at level 24.

2. a. Entering the elevator at level −6 and riding it up 40 floors

b. Entering the elevator at level 24 and going down 28 floors



Hints and Comments

Overview

Students determine how to find the number of the floor in an underground parking garage with floor numbers ranging from −6 to 40. The ground floor is marked 0.

About the Mathematics

In this section, students practice adding and subtracting integers in a more formal way. Throughout the section, the notation changes from informal to formal. On this page, the notation in words, as used in the previous section, is still used.

Planning

You may want to use a number line and refer to Ronnie the Robot for some students. Encourage students who are still counting on the number line to make the calculations in two steps, first from the present floor to the ground floor (0) and next to the floor at which the elevator arrives.

Calculating with Positive and Negative Numbers

Notes

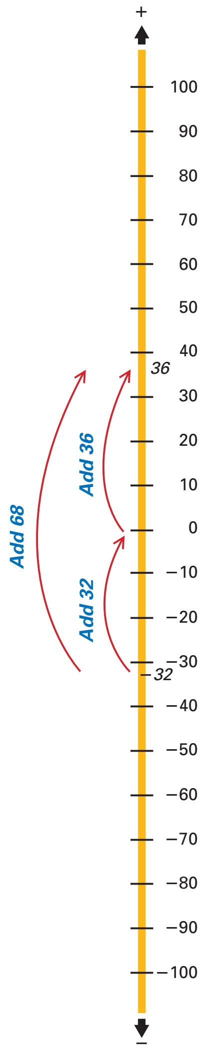
3 This problem may confuse some students. You might want to suggest to students that they are traveling from the lowest floor to the highest floor.

4b Make sure your students relate problem 4b to what they did in problem 4a.

3. a. What is the maximum number of levels that can be covered going up in the elevator?

b. Write a statement using addition that uses this maximum number of levels.

c. Now write a statement using addition that uses the maximum movement down.



Suppose you wanted to do the calculation below. One way is to split the number 68 in two parts, as you can see in the number line to the left. The result is 36.

4. a. How does splitting 68 in this way help you to do the calculation?

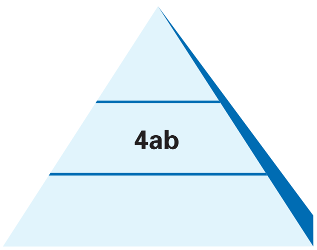
b. How can you change the picture to show the following?

From now on, such calculations will be written in a shorter way.

$(-32) + (68) = (36)$

$(32) + (-68) = (-36)$

Assessment Pyramid



Recognize and use the property of opposites.

Reaching All Learners

Vocabulary Building

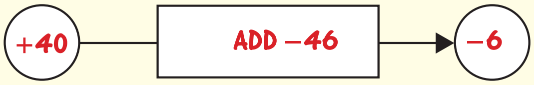
For problem 4, make sure students understand the related terminology that adding a positive makes the elevator go up and adding a negative causes the elevator to go down.

Solutions and Samples

3. a. 46

b.

c.

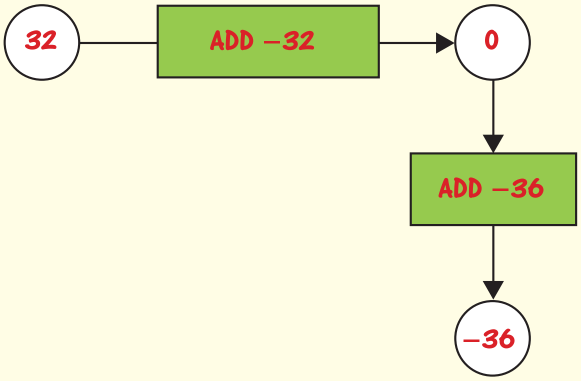


4. a. Move up from −32 to 0 by adding 32, then you still have to add 36 to get a total of 68. You arrive at 36.

Sample student work:

Splitting 68 in this way can help you to do this calculation because you know the two numbers that let you get to 68, which would be 32 and 36. It makes it easier when you break up the number so you can find how many you need to get to zero and then how many you need to get to the number you want.

b.



Sample student work:

32 ADD −68 is −36.

You would have to go −32 to just get to 0 and then go another −36 to get to −36.

Hints and Comments

Overview

Students practice adding and subtracting integers within the context of elevator rides in an underground parking garage.

About the Mathematics

On this page, the informal notation, with the words ADD and SUBTRACT, is still used. The number line model is meant as support for students to break up the calculation in two steps. The number line is in a vertical position to resemble an elevator ride.

Planning

Problem 4 can be done as a whole-class activity.

Calculating with Positive and Negative Numbers

Notes

5 If students are struggling, refer to the elevator idea (e.g., adding a positive makes the elevator go up; adding a negative makes the elevator go down).

Pay close attention to the types of addition problems with which students are successful and those with which they have difficulty. Some students may see a pattern for adding numbers with the same sign (add the absolute value) and for adding numbers with different signs (subtract the absolute value). If students see this pattern, have them share it. However, students should not be forced to memorize this rule at this time.

5. Complete the following calculations. You may draw a number line if it is helpful.

a. (30) + (−60) = ...

b. (32) + (−58) = ...

c. (32) + (−48) = ...

d. (24) + (−48) = ...

e. (48) + (−24) = ...

f. (−30) + (60) = ...

g. (−30) + (−60) = ...

h. (−32) + (35) = ...

i. (−32) + (28) = ...

j. (−32) + (−28) = ...

Reaching All Learners

Advanced Learners

Have some students create and solve problems involving three or more integers rather than just two.

Intervention

Encourage students to use a number line if they are having difficulty with adding these integers.

Solutions and Samples

5. a. $30 + -60 = -30$

b. $32 + -58 = -26$

c. $32 + -48 = -16$

d. $24 + -48 = -24$

e. $48 + -24 = 24$

f. $-30 + 60 = 30$

g. $-30 + -60 = -90$

h. $-32 + 35 = 3$

i. $-32 + 28 = -4$

j. $-32 + -28 = -60$

Hints and Comments

Overview

Students practice addition and subtraction with integers, using a preformal notation.

About the Mathematics

Instead of words, the operator signs + and − are now used. Circles around the numbers are still used to help students view the "operation" sign separate from the positive or negative sign.

Planning

Problem 5 can be assigned as homework.

C Calculating with Positive and Negative Numbers

Notes

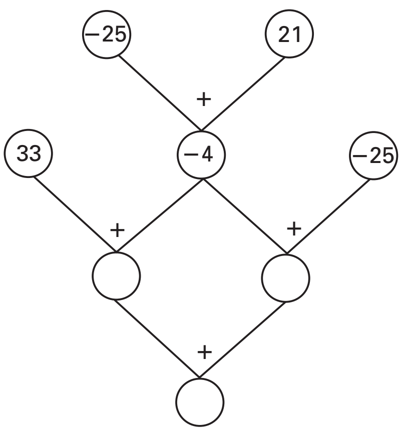
You may want to make an overhead of the trees on this page to demonstrate solutions.

6b This is a more difficult problem because students need to work backwards to complete the tree. You may refer to elevator problems (e.g., What floor do you start on so that if the elevator rises 10 floors, you will end up on floor −25?).

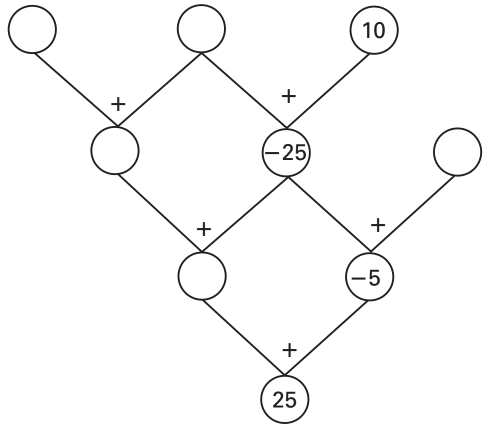
7 Remind students that they may not fill in the tree using just zeroes.

6. Use **Student Activity Sheet 2** to complete the two "adding trees."

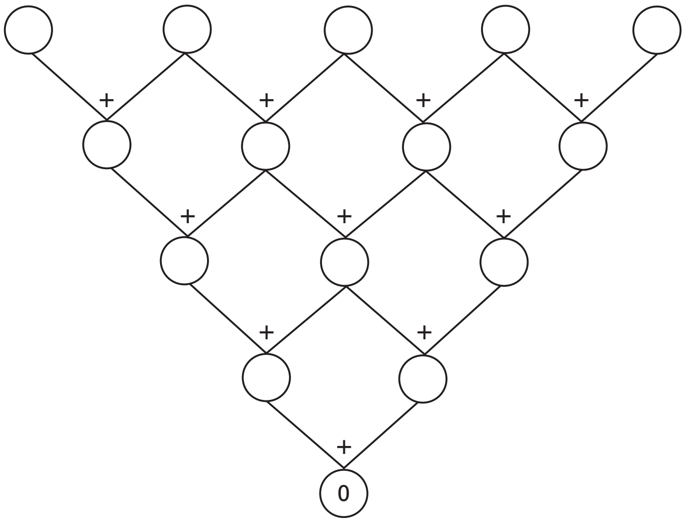
a.



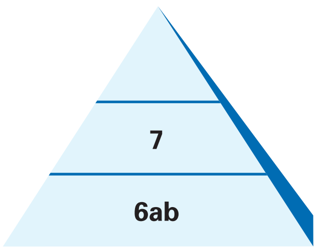
b.



7. There are many ways to fill in numbers in the following adding tree. Copy the table and fill it in so that all of the numbers are different and the final result is 0.



Assessment Pyramid



Recognize and use the property of opposite. Add positive/negative numbers.

Reaching All Learners

Intervention

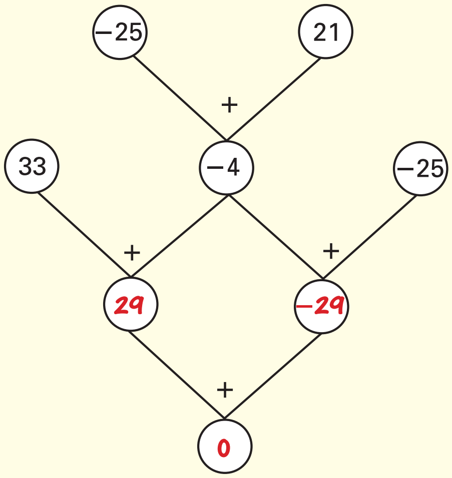
To demonstrate how problem 6a works on the overhead, circle the numbers that you add in the same color and circle the answers in the same color so that students can see how the tree works. As students complete the tree diagram, have them circle the operation.

Advanced Learners

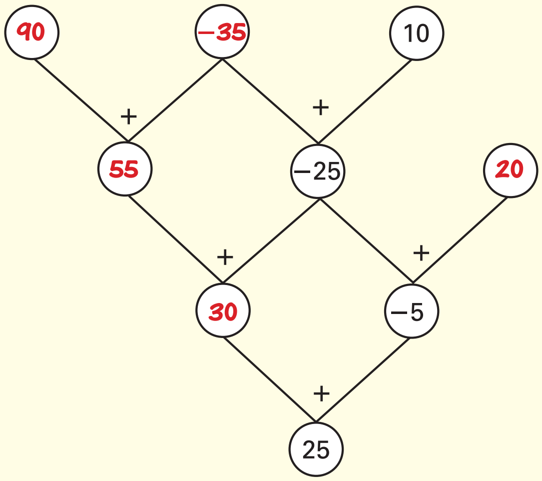
Encourage students who are able to do so to use decimal numbers or fractions for problem 7.

Solutions and Samples

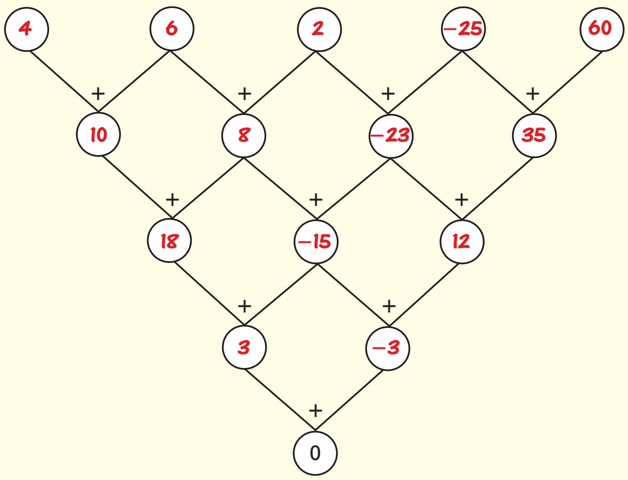
6. a.



b.



7. Answers will vary. Discuss different answers. The final answer should be 0.



Hints and Comments

Materials

Student Activity Sheet 2 (one per student); overhead of trees, optional

Overview

Students practice addition and subtraction with integers, using “addition trees.”

About the Mathematics

The addition trees help students to become flexible in operating with integers. Although the operator signs are always **+**, students need to work “backwards”, thus in fact practicing subtraction.

Planning

Discuss answers with students before they start with problem 7, which is an open problem. This problem provides an opportunity for students to show what they really know and can do because they choose the numbers themselves. You may want to have students copy the tree in their notebooks or provide copies yourself.

Calculating with Positive and Negative Numbers

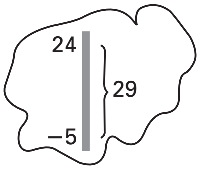
Notes

Encourage students to think of Ronnie the Robot or draw actual number lines if they need additional support thinking through these problems.

10 Make sure that students, in their journals or notebooks, note this statement and other integer relationships discussed on this page.

Recall the building in problem 1.

The distance between the levels 24 and −5 is 29 stories. This is written as:



24 − −5 = 29

This is an example of a subtraction.

8. a. Complete the following subtractions.

i. 14 − −5 = ◯

ii. 4 − −5 = ◯

iii. −4 − −5 = ◯

b. Compare these calculations to:

i. 14 + 5 = ◯

ii. 4 + 5 = ◯

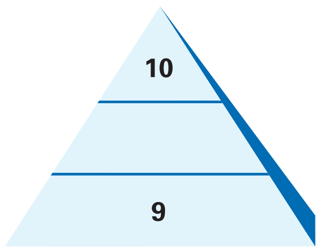
iii. −4 + 5 = ◯

In general, *subtracting* −5 gives the same result as *adding* 5.

9. Give three examples that show that subtracting −10 gives the same result as adding 10.

10. Complete the following statement: *Subtracting 8 gives the same result as adding ___.*

Assessment Pyramid



Generalize rules for operating with positive and negative numbers.

Describe patterns involving integers.

Reaching All Learners

Extension

Give each student two number cubes (1 red and 1 black numbered with 1–6), and one cube with addition and subtraction symbols. A red number cube represents negative numbers, and a black number cube represents positive numbers. Roll the red number cube and the operations cube. Students write "same as" statements. For example, if you roll a "+" symbol and a red 4, students would write that adding a positive four is the same as subtracting a negative four. Repeat 4 times and then switch to the black number cube.

Solutions and Samples

8. a. i. 19

ii. 9

iii. 1

b. i. 19

ii. 9

iii. 1

9. Answers will vary. Sample answers:

$5 - (-10) = 15$ and $5 + 10 = 15$

$-6 - (-10) = 4$ and $-6 + 10 = 4$

$2\frac{3}{4} - (-10) = 12\frac{3}{4}$ and $2\frac{3}{4} + 10 = 12\frac{3}{4}$

10. Subtracting 8 gives the same result as adding −8.

Hints and Comments

Overview

Students review the contexts of the elevator ride to subtract integers in a preformal way. They discover that subtracting an integer is the same as adding the opposite.

About the Mathematics

While playing Ronnie the Robot, students practiced subtraction of a negative number in an informal way. On this page, subtraction is used in a more formal way, with reference to the distance between numbers on the number line. This model still serves as a support for students to strengthen the concept of subtraction. Addition and subtraction of integers is practiced in many different ways before students start with multiplication.

Planning

Students may work on Problems 8–10 in pairs or small groups. Discuss the results afterwards in class.

C Calculating with Positive and Negative Numbers

Notes

11 This is probably the first time students have ever seen a "paired" equation. If students are having difficulty solving these problems, complete one pair of problems using the robot model to show that they arrive at the same answer in both cases.

11 These problems should be used for more than computation practice. Another way for students to look at the "paired" equations is that both of the stacked expressions are equal. For example, subtracting (−10) is the same as adding (10).

11. Complete the calculations.

a. 8 − −10 = ○
8 + 10 = ○

b. −8 + −20 = ○
−8 − ○ = ○

c. −22 + 12 = ○
−22 − ○ = ○

d. 75 − ○ = 99
75 + ○ = 99

Activity

The Integer Game

Here are rules for a card game using 40 index cards. Number 10 cards with consecutive integers from 1 to 10, using a black marker, with one number on each card. Repeat with 10 cards using a red marker, numbering each card with consecutive **integers** from −10 to −1. Make two complete sets of each color. Play with two or three players.

- Shuffle all of the marked cards together.
- Each person gets 7 cards and the rest go face down in a stack in the middle of the table.
- Decide who plays first. The first player uses as many cards as possible that add up to −2 and lays them on the table for others to see.
- Put the cards used with −2 on the bottom of the stack and turn over a new card from the stack.
- The first player tries to use cards that add up to the number on the new card. If that is not possible, it is the next player's turn.
- If the player does not make the number the first time, the player draws a card from the stack. If the player still cannot make the number, it is the next player's turn.
- The first player without any cards wins the game.

Reaching All Learners

Extension

Have students make up their own set of paired equations so they can discover on their own the values that can and cannot be used for a set of circles.

Parent Involvement

Encourage students to play the Integer Game at home with their families.

Solutions and Samples

11. a. $8 - (-10) = 18$

$8 + 10 = 18$

b. $-8 + (-20) = -28$

$-8 - 20 = -28$

c. $-22 + 12 = -10$

$-22 - (-12) = -10$

d. $75 - (-24) = 99$

$75 + 24 = 99$

Hints and Comments

Materials

index cards for the Integer Game (20 per pair or group of three students)

Overview

Students practice addition and subtraction with integers and play the Integer Game. They use the fact that subtracting an integer is the same as adding the opposite.

Planning

Problem 11 may be assigned as homework.

Technology

Instead of the Integer Game, you may also have students use the applet Tic-Tac-Go on the MiC website, mathincontext.eb.com, for extra practice.

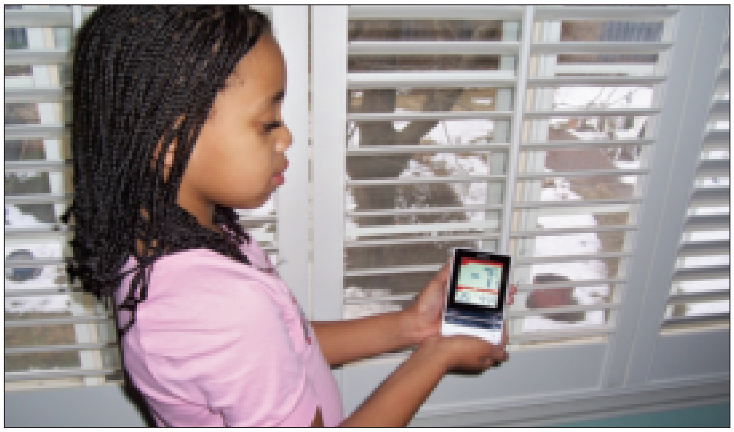
Calculating with Positive and Negative Numbers

Notes

12 Ask students about the Celsius scale. They should know that 0°C is the temperature at which water freezes and that 100°C is the temperature at which water boils.

13 The formal algorithm for division of integers is not being taught here. However, students should be able to determine the correct sign, + or −, from the context. Some students may find that dividing a negative number into parts will result in a negative number.

Temperatures and Altitudes

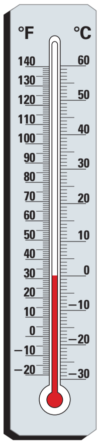


During the winter, Karen's class was given the assignment to record the high and low temperatures for one week.

Here are the temperatures Karen recorded.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
High Temp (°C)	9	3	4	−2	−1	3	−1
Low Temp (°C)	−1	0	−4	−3	−7	−1	−2

12. a. Which day was the coldest? Which day was the warmest? Explain your answers.
    b. Write the low temperatures in order from coldest to warmest. Was the temperature below freezing every day?
13. Calculate the **mean** high temperature for the week. What was the mean low temperature? Show your calculations.



Reaching All Learners

Vocabulary Building

Students learned how to calculate the mean in the unit *Dealing with Data*. Have students share the definition for *mean* in their own words and describe strategies for finding the mean.

Accommodation

Some students may need to use calculators for division when finding the mean.

Solutions and Samples

12. a. Answers will vary. Some students may say that Wednesday had the coldest high temperature of the week, −2°C. Other students may say that Thursday was the coldest day of the week, with a low temperature of −7°C. Most students may say Sunday was the warmest day, with a high of +9°C.

b. −7, −4, −3, −2, −1, −1, 0. It did not get below freezing on Monday.

13. Mean high is 2.14°C.
Mean low is −2.57°C.

Sample calculations:

The mean can be found by adding all the temperatures for the 7 days and dividing both sums by 7.

high temps:

$9 + 3 + 4 + (-2) + (-1) + 3 + (-1) = 15$

low temps:

$(-1) + 0 + (-4) + (-3) + (-7) + (-1) + (-2) = (-18)$

mean high: $15 \div 7 = 2\frac{1}{7}$°C, or 2.14°C

mean low: $-18 \div 7 = -2\frac{4}{7}$°C, or −2.57°C

Hints and Comments

Overview

Students explore a new context (measuring temperatures) in which positive and negative numbers are used.

About the Mathematics

Finding the mean of a data set that contains negative and positive numbers is used to informally introduce multiplication of integers later in the section, when multiplication is performed as a repeated addition. The notation for addition and subtraction is formalized from now on. Parentheses can be used if necessary.

Planning

If necessary, you may want to lead a class discussion on how to calculate the mean. For some students, you may need to review the different temperature scales of Fahrenheit and Celsius.

Comments About the Solutions

13. When discussing this problem, make sure that students are correctly adding positive numbers to negative numbers; for example, they should know now that adding 9 and −2 results in 7, not 11.

Did You Know?

Apart from the Celsius and Fahrenheit scales, there is yet another scale, named after the French scientist René Antoine Ferchault de Reaumur (1683-1757). On the Reaumur scale, the interval between the temperature when water freezes and water starts to boil is divided into 80°, fixing the ice point at 0° and the steam point at 80°. The Reaumur scale is no longer in use.

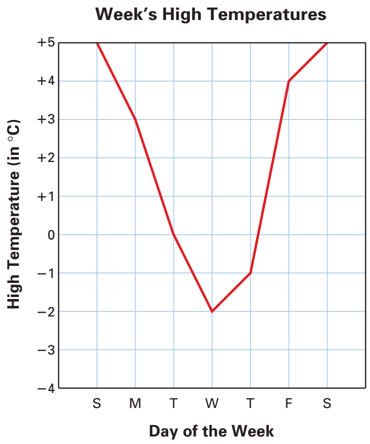
Calculating with Positive and Negative Numbers

Notes

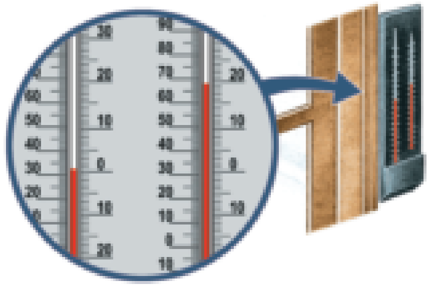
Some teachers may want to ask their students to make both a graph and a table to compare the data of both weeks.

15 Remind students of the "Sun and Snow" problem from the *Dealing with Data* unit. Some students may recognize similar features with this problem.

16 Discuss with students why Diego is canceling out pairs of opposite numbers. Discuss this problem as a whole class to remind students that they are dividing by the total number of values.



The next week, Karen again kept records of high and low temperatures. Instead of writing them in a table, she made a graph. This graph shows the high temperature for each day.



14. **Reflect** Compare the high temperatures from this graph to those from the table for the previous week. Which week was warmer? Give reasons to support your answer.

Karen calculated the mean low temperature in the second week to be −1°C.

15. Come up with one possibility for each of the seven daily low temperatures that week in degrees Celsius.

Here is a list of high temperatures for the whole month of January.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
	−1	+4	0	−1	−1	+1
−1	+3	0	0	−1	+1	+1
−3	−3	−5	−1	0	0	+1
+1	+2	+2	0	−2	−2	−1
−1	+2	+2	−2			

Karen and her classmate Diego both want to calculate the mean high temperature for January. Diego starts crossing out pairs of opposite numbers (for example, +2 and −2) and then adds all of the remaining temperature numbers.

16. By what number does Diego divide to find the mean high temperature?

Reaching All Learners

Accommodation

For problem 14, create a blank table for students, similar to the table for problem 12, and have them fill in the high temperatures for Sunday through Saturday using the graph. Compare the values for each day using both tables.

Advanced Learners

For problem 15, have students create an additional table of values that has a mean of negative 1 degree Celsius or a table of values for a given mean temperature.

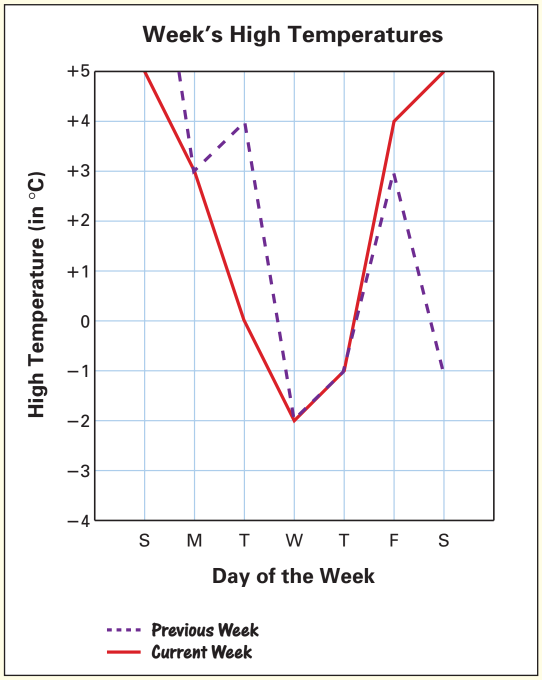
Solutions and Samples

14. Answers and reasoning will vary.
Sample responses:

- The high temperatures are very similar for the middle of the week, but the beginnings and ends are different. There were two days when week 1 was warmer (Sunday and Tuesday) and two days when week 2 was warmer (Friday and Saturday).
- The mean high temperature for week 1 was $2\frac{1}{7}$°C. The mean high for week 2 was 2°C. So you could say that week 1 was slightly warmer.
- Sample table:

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
High Temp (°C) Previous Week	9	3	4	−2	−1	3	−1
High Temp (°C) Current Week	5	3	0	−2	−1	4	5

- Sample graph:



15. Answers will vary. Any set of seven temperatures that add up to −7°C will have a mean of −1°C.

Sample answer sets:

[4, 0, −1, −4, −2, −3, −1]

[2, 2, −1, −3, −3, −2, −2]

16. 31 days

Hints and Comments

Materials

Copies of the graph from Student Book page 29, optional (one per student)

Overview

Students compare temperatures for two weeks using a graph and calculate the mean temperature for a whole month.

About the Mathematics

The mean is a one-number summary of a set of data. The mean does not give any information about the variation in the data. Similarly, means do not indicate which of two sets was more varied.

To compare two sets of data, one can look at the raw data and put these in a chart or line graph. A graph is often very useful for comparison because of the visual aspect. Other features to look at when comparing are the minimum, the maximum, the mode, and the median. These statistical measures were taught in the unit *Dealing with Data*.

Comments About the Solutions

15. Many students will find an "easy" way to solve this problem by recording 0° for six days and –7° for one day, or by recording –1° for seven days. They still discover the important concept, but you may want to encourage them to find other combinations as well. Make sure that the low temperature that students assign for each day is lower than the high temperature for that day shown in the graph.

Calculating with Positive and Negative Numbers

Notes

17a Discuss how Karen initially filled in tally marks (taken from the first week in January calendar).

17b This problem lends itself to a good class discussion.

18 This problem is critical in that students are encouraged to multiply and divide integers using repeated addition or another strategy. Observe how students combine numbers and solve this problem. If they encounter difficulties, encourage them to use repeated addition.

Karen starts by tallying how often each temperature occurred.

−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5
				///	/	/			/	

17. **a.** Explain how Karen should continue in order to calculate the mean.

b. **Reflect** Which method do you like better, Karen's or Diego's? Why?

18. **a.** Copy and finish Karen's tally table.

b. Because there are three days with a temperature of −2°C, Karen calculates:

$$3 \times -2 = -2 + -2 + -2 = \_\_\_\_$$

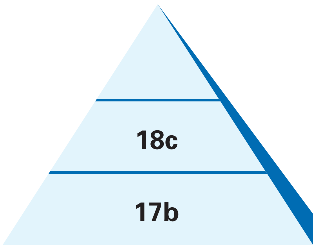
What is the answer? Make calculations like this for each entry in the table.

c. Calculate the mean high temperature.

Higher and Higher

Karen and Diego's school is near a ski area. There are three ski lifts on the mountain. The lowest one is 1,300 m above sea level. From this lift station, a ski lift can take you to the highest lift station at 2,300 m. There is a middle lift station halfway between these two stations.

Assessment Pyramid



Use an illustrative context to help solve problems about integers.
Recognize and use the property of opposites.

Reaching All Learners

Accommodation

For problem 18a, enlarge Karen's tally table and make an overhead. Some students may need calculators for division when calculating the mean in problem 18c.

Intervention

For problem 18b, encourage students to use repeated addition if they have difficulty.

Solutions and Samples

17. a. Karen should finish her tally, multiply each entry by the number of times it occurs, and add all the results. Then she should divide by 31.

b. Answers will vary. Sample response:

Diego's method is faster (especially since some pairs of days, such as Friday and Saturday of the first week, cancel each other out). Karen's method provides a better sense of the distribution of temperatures.

18. a.

Temperatures	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5
Tallies	/		//	///	~~////~~ ///	~~////~~ /	~~////~~	////	/	/	
Count	1	0	2	3	8	6	5	4	1	1	0

b. $3 \times -2 = -6$. Calculations will vary. Sample calculations:

$1 \times -5 = -5$

$0 \times -4 = 0$

$2 \times -3 = -6$

$3 \times -2 = -6$

$8 \times -1 = -8$

$6 \times 0 = 0$

$5 \times +1 = +5$

$4 \times +2 = +8$

$1 \times +3 = +3$

$1 \times +4 = +4$

$0 \times +5 = 0$

c. The mean high is −0.2°C.

The sum of all the temperatures is −5. The mean high temperature is $-5 \div 31 \approx -0.2$°C.

Hints and Comments

Overview

Students explore different ways to calculate the mean temperature if the data set contains both positive and negative numbers. They are introduced to a new context, the elevation of ski lifts on a mountain.

About the Mathematics

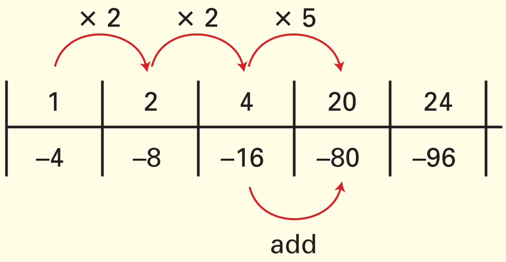
When a series of the same numbers is to be added, one can use repeated addition as a simple form of multiplication. For example:

$4 + 4 + 4 + 4 + 4 = 5 \times 4$

$-4 + -4 + -4 + -4 + -4 = 5 \times -4$

For some students, repeated addition is a more insightful way to multiply numbers.

Note that for larger numbers, a ratio table can be used.



For additional problems that use the ratio table, see the *Number Tools* resource. By working on the problems on this page, students anticipate the use of the double number line, used to explain how integers are multiplied in the next section.

Comments About the Solutions

17. To find the mean, the sum of all temperatures must be divided by 31, the number of days in January.

C Calculating with Positive and Negative Numbers

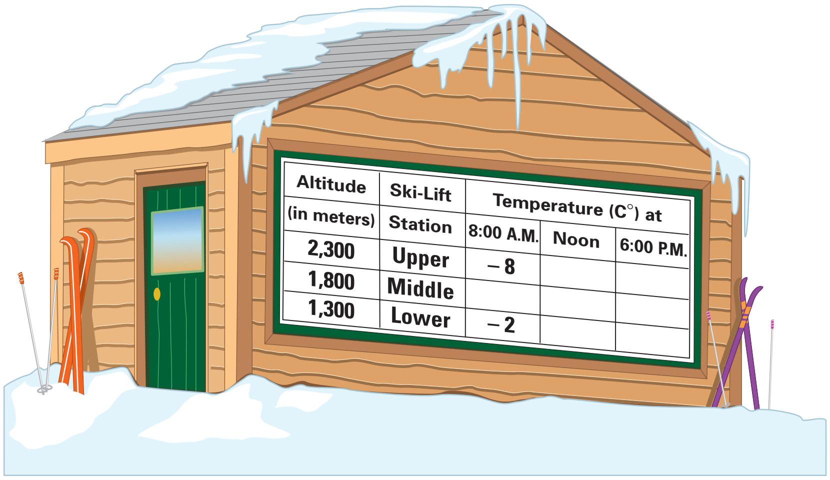
Notes

19a Many students chose negative 6 as the temperature for the middle station. Use this as an opportunity to discuss median.

19b Students often state that the temperature rises by 3°C.

20b Students assume that the temperatures rise another 5°C from noon to 6 P.M. Ask students to consider the temperatures at noon and 6 P.M. on a normal day.

A sign at the lower lift station shows the temperatures at each lift station at different times of the day.



19. **a.** What do you think the temperature at the middle lift station was at 8:00 A.M.?

b. What do you notice about the temperature as you move higher up the mountain?

At noon, the temperatures at all three ski lift stations have risen by 5 degrees.

20. **a.** Write down the noon temperature for each ski lift station.

b. Assume that the differences in temperature between the ski lift stations are always the same. What might the temperatures be at 6:00 P.M.?

In the mountains, a change in height results in a regular change in temperature.

21. In this ski area, what happens to the temperature each time you move up 500 m?

Reaching All Learners

Accommodation

Make copies of the table and distribute one to each student. You may also want to make an overhead of the table, as a visual reference for classroom discussion.

For problem 20, if students have difficulty understanding that the differences remain the same, you may suggest that they use a number line.

Solutions and Samples

19. a. Answers will vary.

Some students will say about −5°C. Strategies will vary. Students may reason that the middle ski lift is halfway between the upper and lower lifts, so its temperature is probably about halfway between those of the upper and lower lifts (−8°C and −2°C).

b. Answers will vary. Most students will say the temperature drops, or gets colder.

20. a. Upper: −3°C

Middle: 0°C

Lower: +3°C

b. Answers will vary. Sample response:

Upper: −5°C

Middle: −2C

Lower: +1°C

Some students will reason that the temperature usually drops by the end of the day.

21. It drops by 3°C.

Hints and Comments

Overview

Students investigate the influence of elevation on temperature.

About the Mathematics

There is a linear relationship between height and temperature; for every 100 m gained in elevation, the temperature drops 0.6°C.

Note that when differences remain the same, the distance between the numbers on the number line remains the same. For example, the distance between −4 and −1 on the number line is 3, and the distance between the numbers 2 and 5 is also 3.

Planning

You may have a brief discussion on how elevation affects the temperature and have students share their experiences. The rule relating temperature and elevation has not yet been made explicit. So if students have not yet discovered a pattern, do not insist that they do so.

Calculating with Positive and Negative Numbers

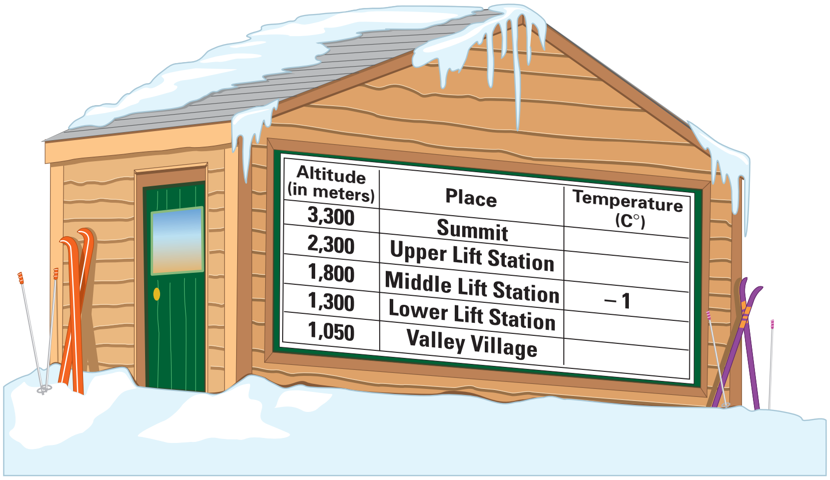
Notes

22 You may need to draw students' attention to the altitude and note that it does not always increase or decrease by 500 m. Also, let students know that the rule applies (that is, the temperature drops as you rise 500 m).

24 You may want to compare the rules students come up with and classify them according to clarity and ease of use.

The next day at noon, the temperature at the middle lift station is −1°C.

22. At noon, what temperature will the sign show for each of the following places?



Here are the high temperatures for a week at the middle lift station.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
High Temp (°C)	−5	−3	−3	0	−4	−3	−5

23. What was the mean high temperature that week at the upper lift station? What was it at the lower lift station?

The general rule you discovered between temperature and altitude tends to be true all over the world: For each 500 m you go up, the temperature drops by about 3°C.

24. a. What happens to the temperature if you move up 1,000 m? 250 m? 100 m?

b. Write a rule describing what happens to the temperature as you go down.

Reaching All Learners

Accommodation

For problem 23, provide students with copy of the chart for High Temp for the middle lift station. However, add a blank row above and below the middle lift row for the upper and lower lift stations. Also, you may want to provide calculators for division when finding the mean.

Solutions and Samples

22.

Altitude (in m)	Place	Temperature (in °C)
3,300	Summit	−10
2,300	Upper Lift Station	−4
1,800	Middle Lift Station	−1
1,300	Lower Lift Station	+2
1,050	Valley Village	+3.5

23. At the upper lift station: −6.3°C, or $-6\frac{2}{7}$ °C

At the lower lift station: −0.3°C, or $-\frac{2}{7}$ °C

Strategies will vary. Sample strategies:

- By subtracting 3°C from each temperature at the middle ski lift, you can find the average at the upper ski lift:

  $-8 + -6 + -6 + -3 + -7 + -6 + -8 = -44$

  The average is $-44 \div 7 = -6.3$°C, or $-\frac{44}{7} = -6\frac{2}{7}$°C
- By adding 3°C to each temperature at the middle ski lift, you can find the average at the lower ski lift:

  $-2 + 0 + 0 + 3 + -1 + 0 + -2 = -2$

  The average is $-2 \div 7 = -0.3$°C or $-\frac{2}{7}$°C
- You can find the average high temperature by (adding 3°C to the middle ski lift average)

  ($-3\frac{2}{7}$°C) to get the lower ski lift average high temperature and subtracting 3°C from the middle ski lift average to get the upper ski lift average high temperature.

24. a. If you go up 1,000 m, the temperature drops by 6°C. If you go up 250 m, the temperature drops by $1\frac{1}{2}$°C, or 1.5°C. If you go up 100 m, the temperature drops by $\frac{3}{5}$ °C, or 0.6°C.

b. Answers will vary. Sample response:

For every 500 m you go down, the temperature rises 3°C.

For every 100 m you go down, the temperature rises 0.6°C.

Hints and Comments

Overview

Students continue investigating positive and negative numbers in the context of elevation and temperature. The rule for the relationship between temperature and elevation is made explicit.

About the Mathematics

The relationship between temperature (in degrees Celsius) and elevation (measured in multiples of 100 meters) can be written as

change in temperature = *change in height* × *− 0.6*

The change in height is positive when going up and negative when going down.

Comments About the Solutions

22. If necessary, refer to the previous problems.

23. It may be valuable to discuss the meaning of −0.3°C, or $-\frac{2}{7}$°C. Students should realize that this is colder than 0°C but warmer than −1°C. You may need to review the < and > signs. If students have problems using fractions, you may want to refresh students' knowledge of the relationship between fractions and decimals as learned, for example, in the unit *Fraction Times*.

Calculating with Positive and Negative Numbers

Notes

25 Remind students that water freezes at 0°C. Suggest to students that they may want to extend the table to fill in additional values.

25 In addition to the answers given, it may be noted that a small amount of snow does evaporate when the sun hits it.

26 For students who do not understand what is being asked of them in this problem, guide them through step by step to explain the meaning of each number in the equation.

You may find heavy snow at higher altitudes. Sometimes snow remains at high altitudes all summer long.

Suppose that the summit of a mountain is 4,418 m above sea level. This mountain descends into the sea. At sea level, the temperature is 23°C.

25. Is the snow melting at the summit? You may continue to build a table like the one below to help you.

Altitude (in m)	0 (sea level)	500	1,000	
Temperature	23°C			

Marcos is working on the following problem.

The temperature at the foot of a mountain is 12°C. What would the approximate temperature be at 300 m up the mountain?

Marcos writes 12 + 3 × −0.6 =

26. Explain how this calculation fits the problem. Then find the answer.

Reaching All Learners

Intervention

For problem 26, to improve student understanding of the connection between contextual information and the equation, have students create their own equation given a different starting temperature and/or rising point.

Solutions and Samples

25. The snow is not melting at the summit since the temperature is below 0°C. The temperature decreases 3°C for every 500-meter increase in elevation.

Altitude (in m)	Temperature (in °C)
0	23°
500	20°
1,000	17°
1,500	14°
2,000	11°
2,500	8°
3,000	5°
3,500	2°
4,000	-1°
4,500	-4°

26. Explanations will vary. Some students may recall that in problem 24a, they found that every time you go up 100 m, the temperature drops 0.6°C. If you go up 300 meters, the temperature change will be

(3 × −0.6) or −1.8°C, and 12 + −1.8 = 10.2°C. So, the new temperature is 10.2°C.

Sample student work:

- This fits the problem because at the foot of the mountain, it is 12°C and you have to go up 300 m on the mountain.
- The 12 is the temperature at the foot of the mountain; the −0.60 is how much the temperature decreases per 100 m. The 3 is from the 300 m. So it all fits together perfectly.
- 12 is the temperature at the base. 3 is the temperature change every 500 m. And 0.6, or $\frac{6}{10}$, of 500 meters is 300. It equals 10.2.
- It fits the problem because 100 m is 0.6°C cooler or warmer, and since there are 300 m you must multiply by 3.

Hints and Comments

Overview

Students do more calculations using the rule relating elevation and temperature.

About the Mathematics

It is assumed here that students know the order of operations (multiplication and division before addition and subtraction) which was introduced in the unit *Expressions and Formulas* and revisited in the unit *Building Formulas*.

Planning

Multiplication has been used earlier in this unit. Some students will still need repeated addition. If students do not know the order of operations, you may want to spend some time on this topic.

Comments About the Solutions

26. From the context, it is clear that the operations should be done in the right order; the starting temperature is 12°C, and 3 times −0.6 is to be added.

C Calculating with Positive and Negative Numbers

Notes

Have students review the information in the Summary in small groups or at home with their parents.

Some teachers may want to note the difference between the use of the − sign as a part of the (negative) number and as an operator.

C Calculating with Positive and Negative Numbers

Summary

In this section, you found that *subtracting* −15 has the same result as *adding* 15.

(18) − (−15) = (33)

(18) + (15) = (33)

You also found that *subtracting* 15 has the same result as *adding* −15.

(18) − (15) = (3)

(18) + (−15) = (3)

If you write these calculations without circling the numbers, use parentheses for the negative numbers: $18 - (-15) = 33$ and $18 + (-15) = 3$. Note: The command about using parentheses may lead to confusion later if they are not used.

A repeated addition can be written as a multiplication; for instance:

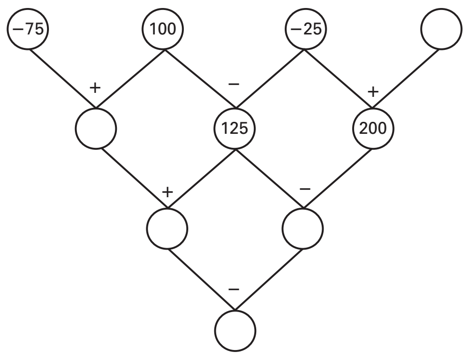
$$(-2) + (-2) + (-2) + (-2) + (-2) = 5 \times (-2) = -10$$

A general rule describing the relationship between temperature and altitude:

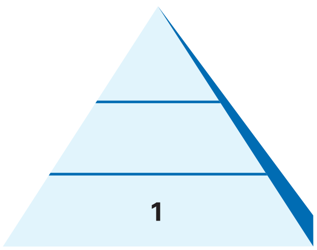
For each 500 m you go up, the temperature drops by about 3°C.

Check Your Work

1. The following tree uses *adding* and *subtracting*. If the sign is −, you have to subtract the number above the sign on the right from the number above the sign on the left. Copy and complete the tree.



Assessment Pyramid



Assesses Section C Goals

Reaching All Learners

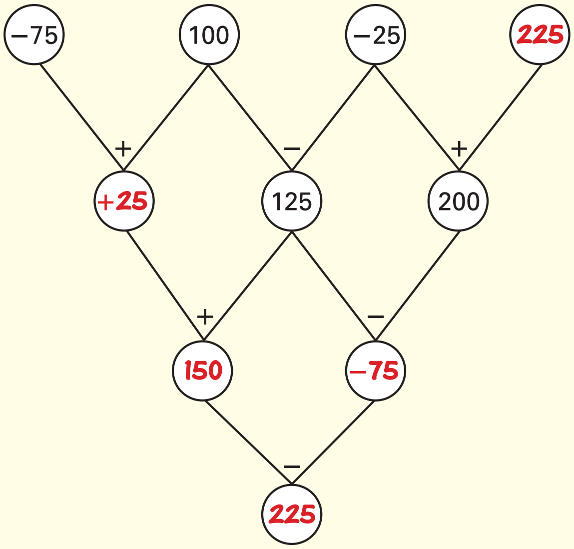
Accommodation

For problem 1, you may want to have students circle the operation as they complete the tree diagram. Make an overhead of the tree diagram and highlight parts of the tree as computations are made, to remind students how to properly complete the tree diagram.

Solutions and Samples

Answers to Check Your Work

1. Remember that if the sign is −, you have to subtract the number above on the right from the number above on the left.



Hints and Comments

Overview

Students read and discuss the Summary, which reviews the mathematical content of Section C. Students use the Check Your Work problems as self-assessment. The answers to these problems are also provided on Student Book pages 60 and 61.

Planning

After students complete Section C, you may assign as homework appropriate activities from the Additional Practice section, located on Student Book pages 54 and 55.

C Calculating with Positive and Negative Numbers

Notes

3 Have students provide examples to support explanations.

2. For ten days, the high temperatures were recorded. On four days, the high temperature was −3°C; on three days, it was −2°C; on one day, it was −1°C; and on two days, it was +2°C.

a. Explain why you can begin to find the mean temperature as follows:

$$(4 \times -3) + (3 \times -2) + (1 \times -1) + (2 \times 2) =$$

b. Finish the calculation for the mean temperature.

3. Which of these four statements will *always* be true? Explain your answer.

a. A positive number added to a positive number results in a positive number.

b. A negative number added to a negative number results in a negative number.

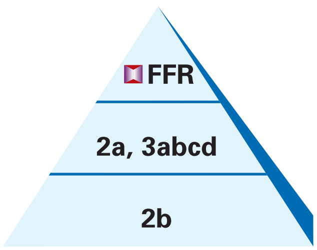
c. A negative number added to a positive number results in a positive number.

d. A negative number subtracted from a positive number results in a positive number.

For Further Reflection

Describe how multiplication is related to addition and how this can help you understand how to operate with negative numbers.

Assessment Pyramid



Assesses Section C Goals

Reaching All Learners

Intervention

For students who are struggling with problem 2, have them use repeated addition to solve this problem.

Extension

Have students create a story involving adding of positive and negative numbers in the context of everyday life (for example, depositing and withdrawing from a checking account).

Solutions and Samples

2. a. Different answers are possible. Sample responses:

There were four -3's, three -2's, one -1, and two 2's. You should multiply each temperature by the number of times that temperature occurred and add the products. Then divide by 10 to find the mean temperature.

b. $(4 \times -3) + (3 \times -2) + (1 \times -1) + (2 \times 2) =$

$$-12 + -6 + -1 + 4 = -15$$

$$-15 \div 10 = -1.5$$

The mean temperature is -1.5°C.

3. a. Always true. If you begin on the right and add a positive number, you move further to the right on the number line. Example: $30 + 60 = 90$

b. Always true. If you begin on the left and add a negative number, you move further to the left on the number line. Example: $-5 + (-10) = -15$

c. Not always true. Example: $6 + (-10) = -4$

Note that in case of a "not always true" statement, you need to give only one "counter-example" to show that the statement is not true.

d. Always true. Subtracting a negative number gives the same result as adding a positive number. So you begin on the right and move farther to the right. Example: $30 - (-20) = 50$

For Further Reflection

If one of the numbers is a whole number, multiplication is the same as repeated addition. So if want to find -3×4, you can add -3 to itself four times. This is like Ronnie the Robot moving backwards 3 spaces 4 times from zero, which is -12.

Hints and Comments

Overview

Students continue working on the Check Your Work problems as self-assessment. The answers to these problems are also provided on Student Book pages 60 and 61.

Section Focus

Addition, subtraction, and multiplication of integers are formalized in this section. Division of integers is informally introduced in this unit and addressed formally in the units *Revisiting Numbers* and *Algebra Rules!* (In algebra units, division of negative numbers is addressed with the computation of *slope*.) In this unit, students anticipate the division of negative numbers as they complete multiplication trees.

Students practice working with integers within the context of finding a mean in a data set. This method of computing the mean for a data set of numbers is a smart way to do so without having to calculate the sum of all numbers. First estimate a mean from the data set. Then find the difference from this mean for each entry and calculate the mean of the differences. (Students could also cancel out opposite differences from the estimated mean.) To find the mean, add (or subtract) the mean of the differences from the estimated mean. The mathematics behind this principle is that when you add (or subtract) the same number to or from all numbers in a data set, the mean changes by the same amount.

Pacing and Planning

Day 12: Calculations Using Differences		Student pages 36–39
INTRODUCTION	Problem 1	Find the mean of a data set using differences from an estimated mean.
CLASSWORK	Problems 3–7	Multiply a negative and a positive number using a double number line.
HOMEWORK	Additional Practice, Section D, Problem 2	Practice finding the mean of a data set using differences from an estimated mean.

Day 13: Multiplication with Positive & Negative Numbers		Student pages 39–43
INTRODUCTION	Problems 8 and 9	Multiply two negative numbers using a double number line.
CLASSWORK	Problems 10–13	Explore a number pattern to support the rules for multiplying integers and complete arithmetic trees for integers.
HOMEWORK	Check Your Work	Student self-assessment: Review topics explored in Section D.

Day 14: Summary		Student page 43
INTRODUCTION	Review homework.	Review homework from Day 13.
ASSESSMENT	Quiz 2	Assessment of Section C and D Goals
HOMEWORK	For Further Reflection	Explain the rules for multiplying integers.

Additional Resources: Additional Practice, Section D, Student Book page 56; *Number Tools*, Section A; *Algebra Tools*, Section A

Materials

Student Resources

Quantities listed are per student.

- **Student Activity Sheet 3**

Teachers Resources

No resources required

Student Materials

No resources required

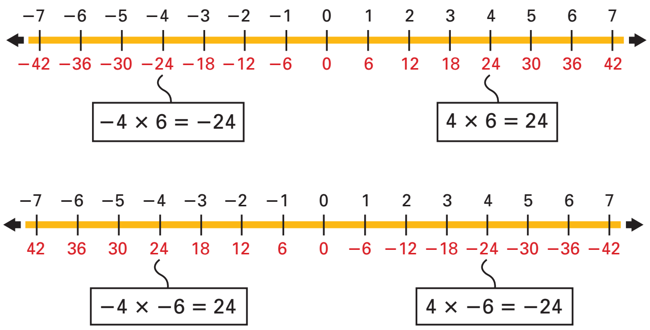
* See Hints and Comments for optional materials

Learning Lines

Number Sense

In order to explain the rules for multiplication with negative numbers, two methods are used, double number lines and the "algebraic principle of permanence."

Double number line:



Algebraic principle of permanence:

For each step, subtract 6	For each step, add 6
$3 \times 6 = 18$	$3 \times -6 = -18$
$2 \times 6 = 12$	$2 \times -6 = -12$
$1 \times 6 = 6$	$1 \times -6 = -6$
$0 \times 6 = 0$	$0 \times -6 = 0$
$-1 \times 6 = -6$	$-1 \times -6 = 6$
$-2 \times 6 = -12$	$-2 \times -6 = 12$
$-3 \times 6 = -18$	$-3 \times -6 = 18$

In this section, the four rules for multiplication are made explicit. Parentheses to discern negative numbers are not used all the time anymore.

At the End of This Section: Learning Outcomes

Students apply and use the formal rules for multiplication of integers. They combine the operations adding, subtracting, and multipling in a variety of problems.

Notes

You may want to list the scores on the overhead so you have them to refer to on the next page.

It might be worth noting to students that the teacher has guessed the mean would be 85. He could have selected any number as the estimated mean.

D Adding and Multiplying

Calculations Using Differences

The scores (in %) on a math test for a group of 20 students are:

74	74	76	80	84
84	84	84	85	88
88	91	91	93	93
93	96	96	97	99

The teacher guesses that the mean will not be far from 85.

To check this, he calculates the difference between each score and 85.

He considers this difference negative if the score is less than 85 and positive if the score is more than 85.

Reaching All Learners

Intervention

You may want to work through the same type of problem using smaller numbers, such as quiz scores from 1 to 10.

Hints and Comments

Overview

Students are introduced to a new method for finding the mean of a large data set. There are no problems for students to solve on this page.

Notes

This method is used as a context to practice working with positive and negative integers.

1c There will be many different strategies used to find the sum of the differences. Be sure to allow the different strategies to be shared with the class.

1c Make sure students record what they found to be the sum before dividing. This allows you to find their errors more easily.

Here is a beginning of the list of differences.

−11	−11	....	....	....
....	....	....	....	....
....	....	....	....	....
....	....	....	....	+14

1. a. Complete the list and add all of the numbers.
   b. Do you think the mean is less than or more than 85? Explain your thinking.
   c. Divide the sum of all of the differences from your list by 20. How can you use this result to find the mean score of the group?

The teacher asked Iris, the student with the highest score, to also calculate the mean.

He did not tell her his result.

Iris guessed the mean would be 90, so she made a list of differences from 90 and calculated the mean score by using the mean of the differences.

2. Make Iris's list of differences and use this list to calculate the mean. Did you find the same result as in problem 1c?

Reaching All Learners

Intervention

In problem 1, have students share where the −11 is coming from. They may need to be reminded that (−) means the score is below the average and (+) means the score is above the average.

Accommodation

You may want to provide a calculator for students to use for division.

Solutions and Samples

1. a.

–11	–11	-9	-5	-1
-1	-1	-1	0	+3
+3	+6	+6	+8	+8
+8	+11	+11	+12	+14

The negative numbers in the list add up to −40. The positive numbers add up to 90. The total is −40 + 90 = 50

b. Discuss answers. Since the total of the negative differences (40) is less than the total of the positive differences (90), the mean will be more than 85.

c. The total of all differences is 50.

50 ÷ 20 = 2.5

The mean score is 85 + 2.5 = 87.5.

2.

-16	-16	-14	-10	-6
-6	-6	-6	-5	-2
-2	+1	+1	+3	+3
+3	+6	+6	+7	+9

The negative numbers in the list add up to –89. The positive numbers add up to 39. The mean is less than 90.

−89 + 39 = −50

−50 ÷ 20 = −2.5

The mean score is 90 + −2.5 = 87.5, which is of course the same answer as in 1c.

Hints and Comments

Overview

Students find the mean of a data set in a smart way, using differences from an estimated mean.

About the Mathematics

The method of computing the mean for a data set of numbers used on this page is a smart way to do so without having to calculate the sum of all numbers. Two advantages of this method are:

- reducing the calculations to smaller numbers, which can often be done mentally, and
- canceling out opposite numbers to reduce the calculations.

First estimate a mean from the data set. Then find the difference from this mean for each entry. Calculate the mean of the differences. Add (or subtract) this mean of the differences from the estimated mean. The mathematics behind it is that when you add (or subtract) the same number to or from all numbers in a data set, the mean will change with the same number.

Note that division by a negative number is not addressed formally in this unit, even though it is indirectly addressed in the context of finding the mean of a set of integers. Formal rules for division of integers are addressed in the units *Graphing Equations* and *Algebra Rules!* where it appears naturally within the concept of *slope*. However, students anticipate division by a negative number as they fill in numbers in the multiplication trees, which is another way to practice carrying out operations with integers.

Planning

You may want to do problem 1 as a whole-class activity and have students work on problem 2 on their own. Problem 2 may also be assigned as homework.

D Adding and Multiplying

Notes

3 Discuss the fact that 4 × −2 means 4 sets of −2. This leads to the discussion of − 4 × 6 later.

4 Students often relate this double number line to a ratio table. Allow this as a possible strategy.

Multiplication with Positive and Negative Numbers

4 times 6 means 6 + 6 + 6 + 6, and the result is 24.

What does *4 times −2* mean?

In Section C, you met Diego, who calculated:

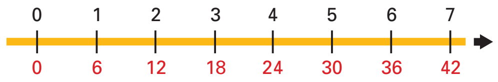
$$4 \times -2 = -2 + -2 + -2 + -2 = -8$$

In the list of differences (see problem 2), you found 2 times −16, 4 times −6, and 2 times −2.

3. Use these numbers to write three calculations like Diego did with 4 × − 2.

You have seen some examples of multiplying a positive times a negative. But how would you think about −4 times 6 or, even worse, −4 times −6? What can be the meaning of *−4 times* something?

In mathematics, it is possible to do multiplications for those two examples. First look at the half number line.



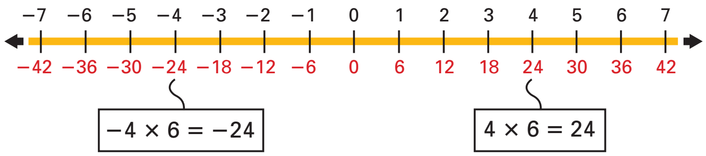
The line has a double scale. The bottom numbers are multiples of 6.

The picture shows, for instance:

$$4 \times 6 = 24$$

4. Suppose that this line is continued to the right. Which number will be just above 84? Above 420? Write the corresponding multiplication.

Now continue the double number line to the left.



Reaching All Learners

Intervention

Although precision is not necessary in marking the values of the double number line, you may want to supply students graph paper or have them use lined paper on its side (i.e., $11 \times 8\frac{1}{2}$ inches).

Parental Involvement

Have students explain to their parents why multiplication is like repeated addition.

Solutions and Samples

3. Answers may vary. Sample student answer:

$2 \times -16 = -16 + -16 = -32$

$4 \times -6 = -6 + -6 + -6 + -6 = -24$

$2 \times -2 = -2 + -2 = -4$

4. Explanations may vary. Possible answers:

84 is double 42, so the black number above 84 is $2 \times 7 = 14$.

This means that $14 \times 6 = 84$.

420 is 10 times 42, so the black number above 420 is $10 \times 7 = 70$.

So $70 \times 6 = 420$.

Hints and Comments

Overview

Students are introduced to the multiplication of a negative and a positive number using a double number line.

About the Mathematics

Multiplication as repeated addition is reviewed. The number line is used as a model once again, now to introduce the concept of multiplication of a negative number by a positive number, which does not occur within a realistic situation. Note that formal notation is used right away.

Planning

Students may work on problems 3 and 4 in pairs or small groups.

Notes

5a Break this down to "take away 10 sets of positive 6." This allows students to see why you end up with a negative answer.

7b Students often confuse the subtracting 11 with negative 11. In this table, students are subtracting 11 as they move from row to row. Make sure this is clear before they begin.

8a Writing the multiplication statement helps to reinforce the concept of multiplication of integers.

The negative numbers are called *negative multiples* of 6.

5. a. **Reflect** How can you explain that −10 × 6 = −60?

   b. Do you think 6 × −10 will have the same result? Why or why not?

6. a. Make a double number line in such a way that the black 1 corresponds to the red 8.

   b. Write three multiplications of the form …. × 8 = …., using negative numbers.

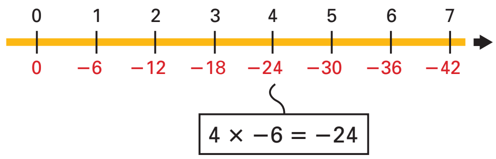
7. a. Complete the multiplication table for 11.

   b. What does the −11 by the arrow in the table have to do with these multiplications?

3 × 11 = 33 ⟩ −11
2 × 11 = 22 ⟩ −…
1 × 11 = 11 ⟩ −…
0 × 11 = 0 ⟩ −…
−1 × 11 = … ⟩ −…
−2 × 11 = …
−3 × 11 = …

Now consider the multiplication of negative numbers.

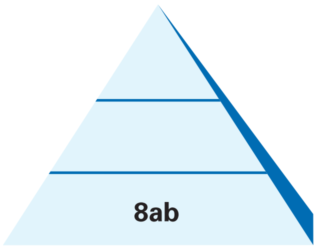
Use a double number line. The numbers are multiples of −6.



8. a. First look at the number line above. Which number will be below 8? Below 12? Write the multiplication statement for each of these.

   b. Which number will be above −60? Above −300?

Assessment Pyramid



Describe patterns using positive and negative numbers.

Reaching All Learners

Intervention

Student desk tape number lines may prove to be useful to students who are having trouble understanding the problems.

Visual Learners

Have an overhead number line available for this problem so that students can demonstrate the jumps along the number line as they work down the table.

Solutions and Samples

5. a. Explanations will vary. Sample explanations:

You may extend the number line.
− 48 − 54 − 60

The right part of the double number line is the opposite of the left part. Since 10 × 6 = 60, on the opposite side you will find − 10 × 6 = − 60.

5 × −2 = −10 The red number corresponding with − 2 is −12 , so the red number corresponding with − 10 is 5 × − 12 = − 60.

Sample student work:

You can say that you have −10 groups of 6 (Like taking away 10 groups of 6), which gives you a total of −60.

What you could do is look for −10 on the number line, then look under the line, you'll see −60.

I can explain it by: −10 is equal to taking away ten multiples of 6. So that means 6 negative tens equals −60.

b. Explanations will vary. Sample explanations:

In multiplication, you know that for instance 5 × 3 has the same result as 3 × 5. Similarly, − 10 × 6 has the same result as 6 × −10.

6 × −10 means −10 + −10 + −10 + −10 + −10 + −10 = −60.

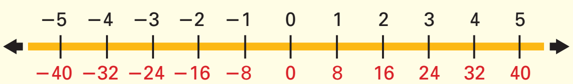
Ronnie the Robot makes 6 jumps. Because the number –10 is a negative number, he starts looking in the direction of the negative numbers.

Sample student work:

Yes, because all you are doing is flipping the numbers, but in subtraction you cannot flip the numbers.

Yes, it will have the same result because 6 multiples of –10 would end up as –60; it's just switched.

6.



Answers will vary. Sample answers:

−3 × 8 = −24

−10 × 8 = −80

−2 × 8 = −16

7. a. −1 × 11 = −11

−2 × 11 = −22

−3 × 11 = −33

b. If you make a multiplication table of times eleven:

1 × 11 = 11,

2 × 11 = 22,

3 × 11 = 33,

4 × 11 = 44,

5 × 11 = 55, and so on, you see that for each step, 11 is added to the previous answer.

In problem 7a, you go "backwards" in the multiplication table, so now for each step, 11 is subtracted (or −11 is added).

8. a. The number below 8 will be − 48, and the number below 12 will be −72.

Sample student work: −6 × 8 = −48, and −6 × 12 = −72.

b. The number above −60 will be 10, and the number above −300 will be 50. Sample student work:

10 × −6 = −60 OR −60 ÷ −6 = 10

50 × −6 = −300 OR −300 ÷ −6 = 50

Hints and Comments

Overview

Students continue exploring multiplication of integers using a double number line.

About the Mathematics

Apart from the double number line, on this page the "algebraic principle of permanence" is used to explain why −4 × 6 = −24

−3 × 6 = −18

−2 × 6 = −12

−1 × 6 = −6

0 × 6 = 0

The first term in the multiplications is descending by one, so the next term should be −1. The second term is kept the same (× 6). The numbers in the result are descending by 6, so the next number is −6 (i.e., −1 × 6 = −6).

Planning

Students may work in pairs or small groups on Problems 5–8. These problems may also be assigned as homework.

D Adding and Multiplying

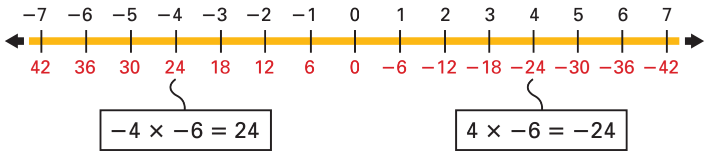
Notes

9 and 10 are practice questions that relate directly to the previous page.

11 This question also addresses students' number sense. Have them share their strategies. This question also provides an opportunity for students to discuss any rules they may have learned when dealing with multiplication of integers.

In the number line below, you see the continuation of the numbers to the left.

Reading from left to right, the numbers below the line are going down, but the numbers above the line are going up.



So you can see that the negative multiples of −6 are positive!

9. **a.** Which number corresponds to −9? Write the multiplication statement.

b. Which number corresponds to 66? Write the multiplication statement.

10. **a.** Complete the multiplication table shown below for −8.

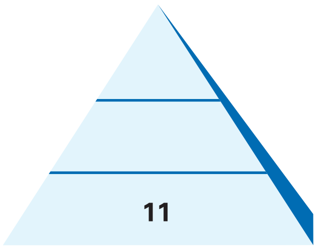
b. What does the +8 by the arrow in the table have to do with these multiplications?

3 × −8 = −24	
	+8
2 × −8 = −16	
	+....
1 × −8 = −8	
	+....
0 × −8 = 0	
	+....
−1 × −8 = ...	
	+....
−2 × −8 = ...	
	+....
−3 × −8 = ...	

11. Complete the following calculations.

20 × −15 =	30 × 5 =	10 × 25 =
20 × 15 =	30 × −5 =	0 × 15 =
−20 × 15 =	30 × −15 =	−10 × 5 =
−20 × −15 =	30 × −25 =	−20 × −5 =

Assessment Pyramid



Describe patterns using positive and negative numbers.

Reaching All Learners

Extension

Discuss the relationship between multiplication and division. Division is not formally covered in this book but is addressed on the unit test. For some students, you may want to refer to division as the inverse operation of multiplication with the same general rules.

Solutions and Samples

9. a. The number below −9 will be 54.

Student sample work:
$-9 \times -6 = 54$

b. The number above 66 will be −11

Student sample work:
$66 \div -6 = -11$

10. a. $-1 \times -8 = +8$

$-2 \times -8 = +16$

$-3 \times -8 = +24$

b. If you make a multiplication table of times negative 8:

$1 \times -8 = -8$,

$2 \times -8 = -16$,

$3 \times -8 = -24$,

$4 \times -8 = -32$,

$5 \times -8 = -40$, and so on, you see that for each step, −8 is added to the previous answer or 8 is subtracted.

In problem 10a, you go "backwards" in the multiplication table. So for each step, −8 is subtracted (or 8 is added).

11. $20 \times -15 = -300$

$20 \times 15 = 300$

$-20 \times 15 = -300$

$-20 \times -15 = 300$

$30 \times 5 = 150$

$30 \times -5 = -150$

$30 \times -15 = -450$

$30 \times -25 = -750$

$10 \times 25 = 250$

$0 \times 15 = 0$

$-10 \times 5 = -50$

$-20 \times -5 = 100$

Hints and Comments

Overview

Students continue doing calculations with positive and negative numbers.

About the Mathematics

The double number line as well as the "algebraic principle of permanence" are used to show that a negative number times a negative number results in a positive number.

Planning

Students may work in pairs or small groups on problems 9 and 10. Problem 11 may be assigned as homework.

D Adding and Multiplying

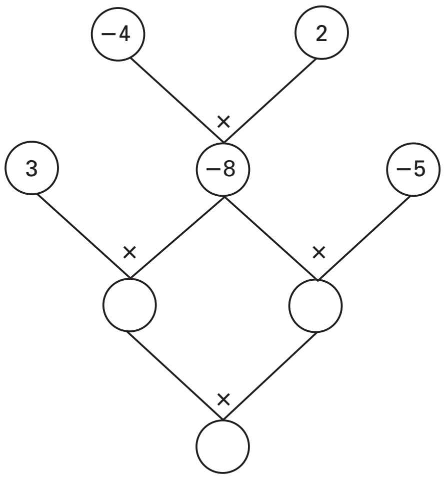
Notes

12 and 13 Make overheads to show solutions.

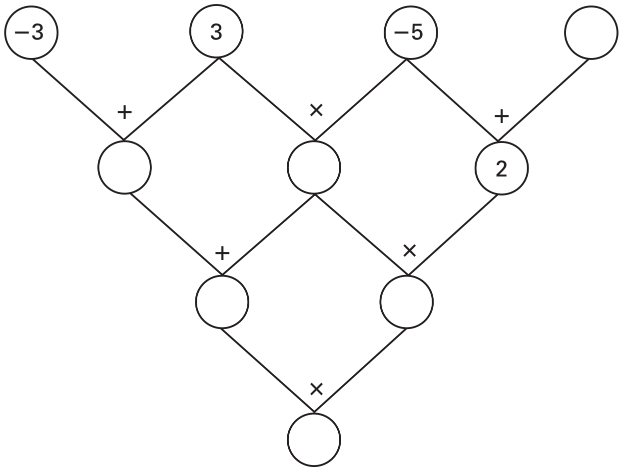
Have students circle the operations. This helps draw their attention to the different operations being used.

Use **Student Activity Sheet 3** for problems 12 and 13.

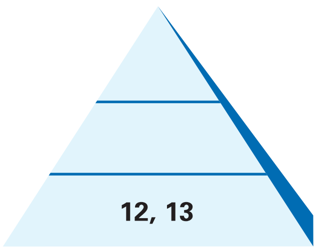
12. Complete the multiplication tree.



13. Here you see a tree with different kinds of operations (+ and x). Complete this tree.



Assessment Pyramid



Perform operations with positive and negative numbers

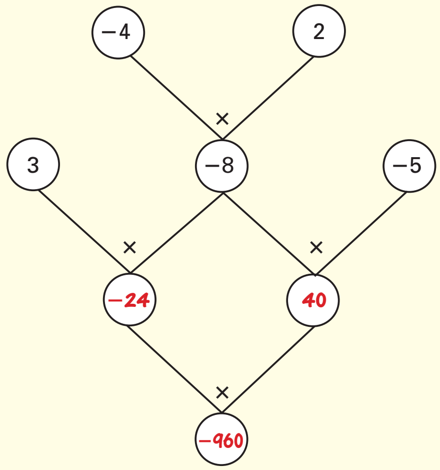
Reaching All Learners

Advanced Learners

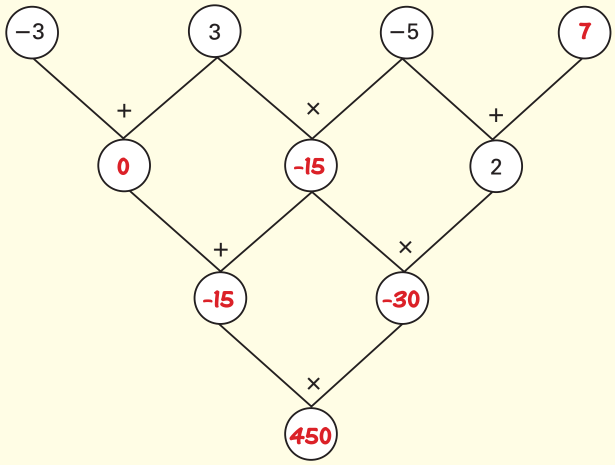
Have students create their own tree and then exchange with others to solve. You may want to challenge students to include benchmark fractions and decimals with their trees.

Solutions and Samples

12.



13.



Hints and Comments

Materials

Student Activity Sheet 3 (one per student)

Overview

Students practice multiplication and addition of integers using multiplication and addition trees.

About the Mathematics

The trees are just another way to help students practice calculations with integers to become more flexible.

Planning

Problems for extra practice can be found in the *Algebra Tools* resource.

Technology

Students may also work with the applet Tic-Tac-Go, which can be found on the MiC website, mathincontext.eb.com.

Notes

Point out that the rules listed for multiplication on this page also apply for division problems. You could have students try and find an example that would disprove one of the rules listed.

D Adding and Multiplying

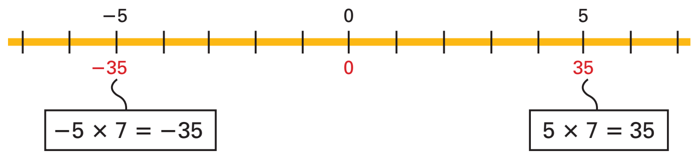
Summary

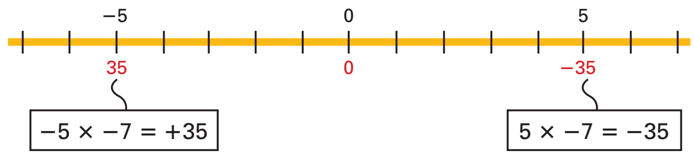
Multiplication with a positive whole number is the same as a repeated addition; for instance:

$5 \times 7 = 7 + 7 + 7 + 7 + 7 = 35$

$5 \times (-7) = (-7) + (-7) + (-7) + (-7) + (-7) = -35$

With the help of a pattern on the number line, you can find the results of multiplying by a negative whole number; for instance:





There are four rules for multiplication of integers.

positive × ***positive*** = ***positive***

positive × ***negative*** = ***negative***

negative × ***positive*** = ***negative***

negative × ***negative*** = ***positive***

Reaching All Learners

Extension

Have students write their own problems multiplying positive and negative numbers. Of course they should supply answers to the problems they made.

Act It Out

This would be a good opportunity to have some students model the four rules for multiplication using a number line on the board or overhead. Many students would gain more from seeing these four rules demonstrated once again, rather than simply reading through the Summary.

Hints and Comments

Overview

Reading the Summary, students review the formal rules of multiplication with positive and negative numbers.

Planning

After students complete Section D, you may assign as homework appropriate activities from the Additional Practice section, located on Student Book page 56.

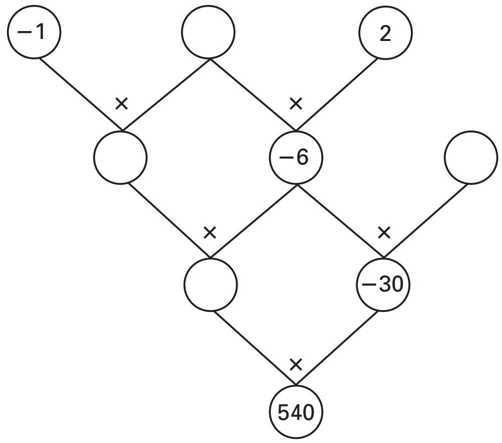
Notes

For Check Your Work, have students discuss strategies for solving all of these problems.

2 Tell students they will need to pick their own test score to use when finding the differences.

Check Your Work

1. Copy and complete the multiplication tree.



2. Calculate the mean score of a test with the following results. Use a table of differences.

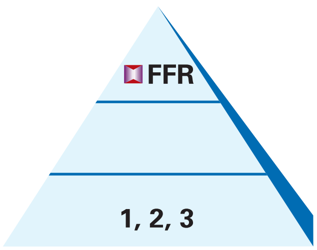
66	68	74	75	75	75
77	77	77	79	80	81
82	83	83	83	85	85
91	95	97	98	100	100

3. Use all three of the operations adding, subtracting, and multiplying and at least two negative numbers to make a calculation string for each of the numbers from −1 to −10 (for example, $(-2 \times 5) - (-4) + 5 = -1$).
Have a classmate check your answers.

For Further Reflection

Write a letter to a friend explaining the rules for multiplication of positive and negative numbers.

Assessment Pyramid



Assesses Section D Goals

Reaching All Learners

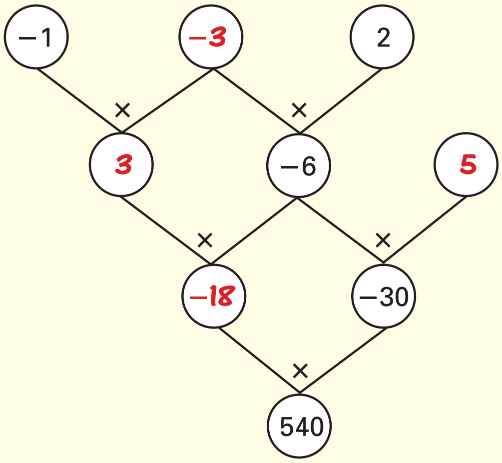
Accommodation

For problem 3, you may choose to replace "calculation string" with "arrow string." Arrow string is a term and method for representing expressions that students should be familiar with from *Expressions and Formulas* and other MiC units.

Solutions and Samples

Answers to Check Your Work

1\.



2\. First make a list of differences from your estimated mean. Suppose you estimated the mean to be 80.

-14	-12	-6	-5	-5	-5
-3	-3	-3	-1	0	+1
+2	+3	+3	+3	+5	+5
+11	+15	+17	+18	+20	+20

Use any method to calculate the total of all differences: +66

You know now that the mean is above 80 since +66 is positive.

$66 \div 24 = 2.75$

The mean score is $80 + 2.75 = 82.75$.

3\. Have a classmate check your answers. If you do not agree, ask your teacher.

For Further Reflection

Answers will vary. The four rules for multiplication must be mentioned in the letter.

Hints and Comments

Overview

Students use the Check Your Work problems as self-assessment. The answers to these problems are also provided on Student Book pages 61 and 62.

Section Focus

Informal introduction of the coordinate system — only positive numbers — has taken place in the unit *Expressions and Formulas*. Students review the positive coordinate system by using the map of Provo, Utah, from *Figuring all the Angles*, to give greater visual coherence between units. In section E, the coordinate system is expanded with negative numbers. Students plot and interpret points on this coordinate system. They practice using the rules for addition, subtraction, and multiplication of integers within the context of transformations of geometric shapes on the coordinate system.

Pacing and Planning

Day 15: Directions		**Student pages 44–47**
INTRODUCTION	Problems 1 and 2	Relate directional coordinates to the standard notation for (x, y) coordinates.
CLASSWORK	Problems 3–6	Review features of a coordinate system and methods for plotting points.
HOMEWORK	Problem 7	Plot and label points on a coordinate system.

Day 16: Changing Shapes		**Student pages 47–48**
INTRODUCTION	Problem 8	Plot points on a coordinate system.
CLASSWORK	Problems 9–12	Predict what will happen to a figure when the coordinates are changed.
HOMEWORK	Problem 13	Describe the effect of multiplying the coordinates of a triangle by a whole number.

Day 17: Changing Shapes (Continued)		**Student pages 48–51**
INTRODUCTION	Problem 14	Reflect on the effect of a negative number on the shape of a figure in the coordinate plane.
CLASSWORK	Problems 15–19	Investigate the effect of multiplication on the coordinates of given figures.
HOMEWORK	Check Your Work, For Further Reflection	Student self-assessment of Section E Goals

Day 18: Summary		
INTRODUCTION	Review homework.	Review homework from Day 17.
REVIEW	Additional Practice or Section Summaries	Review section or unit goals.

Additional Resources: Additional Practice, Section E, Student Book page 57; *Number Tools,* Section A; *Algebra Tools,* Section A

Materials

Student Resources

Quantities listed are per student.

- **Student Activity Sheet 4**
- **Student Activity Sheet 5** (for Additional Practice, Section E)

Teachers Resources

No resources required

Student Materials

Quantities listed are per student.

- graph paper (at least 3 sheets)
- ruler or straightedge

* See Hints and Comments for optional materials

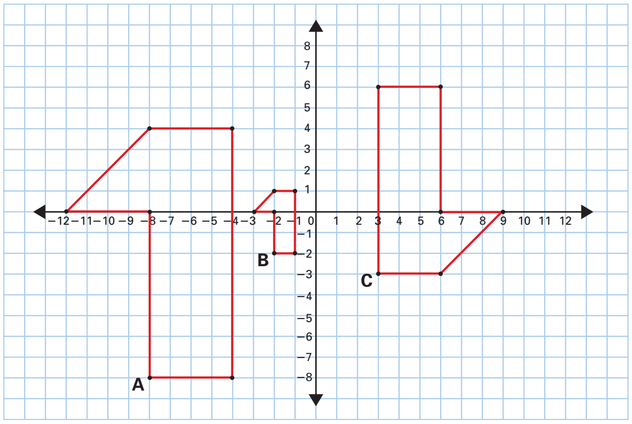
Learning Lines

Number Sense

Students apply operations with positive and negative numbers. By using transformations of geometric shapes on the coordinate system, they find that *negative* × *negative* = *positive* within a new context.

Models

Operations with integers are applied within the model of the coordinate system. A coordinate system is used to locate points on a grid by indicating two coordinates that tell where the point is located relative to the origin.



The position of a geometric shape, drawn on a grid, can be changed by adding a number to or subtracting a number from each coordinate (or one of them). Multiplying or dividing the coordinates by some number is also possible. Students need to apply the rules for the operations with integers to do this, reinforcing their knowledge of the concept of integers. Visualization of the effects of performing operations with integers is a powerful way to understand the concept.

At the End of This Section; Learning Outcomes

Students are able to plot and read points from a coordinate system with four quadrants. They apply the operations with positive and negative numbers by investigating transformations of geometric shapes on the coordinate system.

Notes

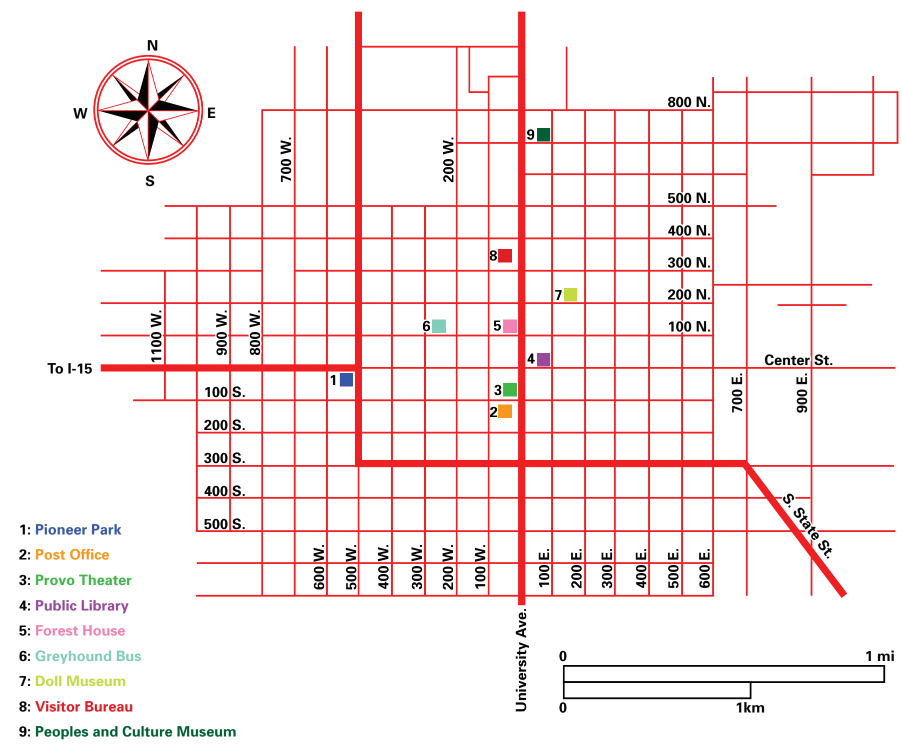
Providing a copy of this page to write on can be useful to students when trying to follow the directions. It is also helpful if they mark the location of Diego's house for future use. Also use a transparency of this so you can point out where things are located.

E Operations and Coordinates

Directions

Diego invites some friends to his home for a sleepover. "It's easy to find," he says. "From Pioneer Park, you go three blocks east and then two blocks south. There is my house."

Downtown Provo, Utah



Reaching All Learners

Advanced Learners

Have students select other locations and write directions to get to and from those locations.

Hints and Comments

Overview

Students investigate the coordinate system.

About the Mathematics

In the unit *Expressions and Formulas*, students worked with coordinate systems and graphs. Only positive numbers were used. In units that follow *Operations*, such as *Graphing Equations* and *Algebra Rules!*, students will use the formalized coordinate system to draw and interpret graphs. This section also shows the connection between operations and some transformations.

In this section, formal mathematical language is introduced: coordinate system, coordinates, origin, and horizontal and vertical axes. The coordinate system is extended to all four quadrants. We do not use the word *quadrant* in the Student Book.

The context used in this part of the section is introduced by using the map of downtown Provo, Utah, from *Figuring All the Angles*, to provide some visual and contextual coherence between units.

E Operations and Coordinates

Notes

When students are working on these problems, make sure that they interpret the coordinates correctly; the first number refers to the left/right direction, and the second number refers to the up/down direction.

1 Students sometimes count up using the blocks instead of the grid lines and end up with (E6, N8) as their answer rather than (E6, N7). Emphasize that the lines are the streets on which people are traveling.

2 Students forget where Diego's house is located, so make sure they marked their answer down from the previous page.

3 This question really depends on what type of area you live in. For those living in a rural area, you may want a map of a nearby city to show that these directions could be used on that map.

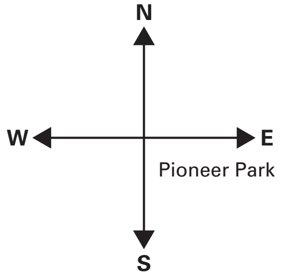
1. The directions Diego gave can be noted as (E 3, S 2).

a. Use the same notation to give directions from Pioneer Park to the Peoples and Culture Museum.

b. Use the same notation to give directions from the Public Library to the Peoples and Culture Museum. Start with the direction for east or west and then state the direction for north or south.

Now that Diego has learned about positive and negative numbers, he decides to change his system. "If Pioneer Park is my starting point, I can use positive numbers for going east and negative numbers for going west, and then positive numbers for going north and negative numbers for going south—just like using two number lines that are perpendicular."

Here is Diego's sketch.



2. Use the new notation to give directions from Pioneer Park to Diego's house.

3. a. What directions does (+2, −1) mean in Diego's notation?

b. What does (0, 0) mean in Diego's notation?

4. Could you use (−8, −5) to give directions in your town? Explain why or why not.

Reaching All Learners

Advanced Learners

Have students use coordinate directions to move from school to a town landmark.

Extension

Have city maps on hand and have students practice giving directions using N, S, E, W, and the number of blocks.

Solutions and Samples

1. **a.** (E 6, N 7)

   b. (E 0, N 7)

2. (+ 3, − 2)

3. **a.** Two blocks East, one block South, starting from Pioneer Park.

   b. (0, 0) in Diego's notation means that he stays at Pioneer Park.

4. It would mean eight blocks west and five blocks south. However, you would not know where to start if you only got these directions, so you cannot really use the directions in your city unless the starting point is also included.

Hints and Comments

Overview

Students use positive and negative numbers to show directions on a map.

About the Mathematics

The vertical and horizontal axes can be seen as a set of perpendicular number lines.

Planning

Problem 1 is used to show the need of a generally used system to locate points on a grid.

Comments About the Solutions

4. Discuss students' answers in class. Diego's notation can only be used in his situation; a more general system is necessary.

Operations and Coordinates

Notes

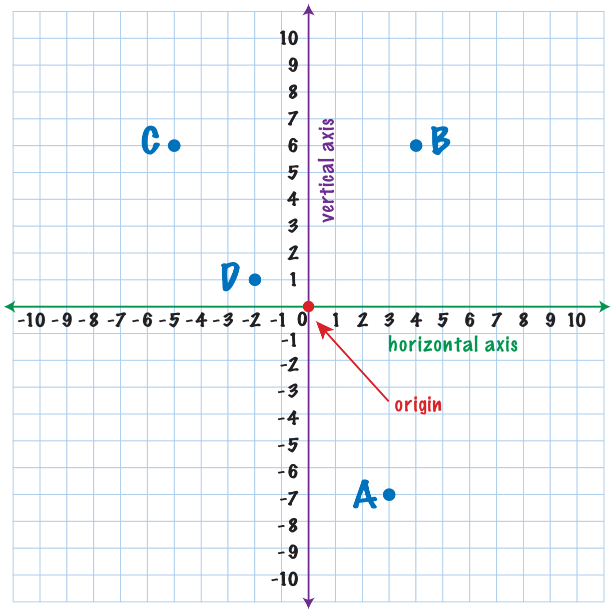
The first coordinate is always the horizontal coordinate, and the second coordinate is always the vertical coordinate. A handy way to remember this is that *x* comes before *y* in the alphabet. This alphabetical ordering is one of the reasons a coordinate is often called an ordered pair.

5 Have an overhead of this grid available for discussion purposes.

Changing Shapes

To communicate about the locations of points on a grid, it is useful if everybody uses the same language and notation. You need to choose a starting point first.

Mathematicians and scientists use a grid with a starting point called the **origin** and a **horizontal** and **vertical axis** with numbers very much like horizontal and vertical number lines. This type of grid is called a **coordinate system**. Positive and negative numbers on the axes can be extended as far as you wish.



On the coordinate system, the **coordinates** for point *A* are written as (3, −7). Note that you start with the horizontal direction and start counting at (0, 0), the *origin*. The + sign for a positive number is usually not shown; it is understood.

5. a. How would you describe the location of points *B* and *C*?

b. Why does it make sense to use the coordinates (0, 0) for the origin?

6. Is (−2, 1) the same point as (1, −2)? Explain your reasoning.

Reaching All Learners

Intervention

For problem 6, if you get many students stating that these are the same point then make sure you demonstrate on the overhead how to find where each of these dots is located. Also give them a few extra practice points.

Extension

Ask students to draw figures of their own on a coordinate system, label the points of the figures, and write the coordinates of each point. Some students may want to start with a pattern of coordinates and draw the corresponding figure.

Solutions and Samples

5. a. *B* (4, 6) and *C* (−5, 6)

b. Because it is the location of the "middle" of both number lines and zero is a good starting point when working with number lines.

6. No, (−2, 1) is not the same point as (1, −2). Sample explanations:

The first number of the pair shows the number of units you go left or right, starting from the origin. So for (−2, 1) you go two units left, and for (1, −2,) you go 1 unit right, starting from the origin.

The first number in the pair is the number of steps on the horizontal axis. — positive number for right, a negative number for left. The second number in the pair is the number of steps on the vertical axis. — positive number for up, a negative number for down.

Hints and Comments

Materials

graph paper (one sheet per student)

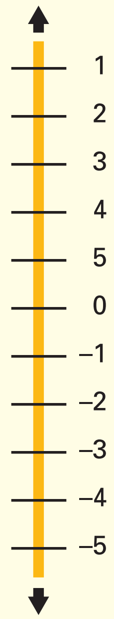
Overview

Students plot and label points on a coordinate system.

About the Mathematics

The horizontal axis is called the *x*-axis, and the vertical axis is called the *y*-axis. A point on the coordinate system has coordinates (*x*, *y*). Points in a coordinate system are, by convention, labeled with capital letters.

A common misconception is placing the numbers on the vertical axis incorrectly, such as:



E Operations and Coordinates

7. Draw your own coordinate system. Use graph paper.
   a. Draw vertical and horizontal axes at least 10 units long. Put a number scale on each axis and a 0 for the origin.
   b. Plot the following points in your coordinate system.

   (1, 1), (3, 3), (2, −1), (−2, −1), (−3, 1), (−4, 0), (−3, 2), (−2, 2), (−1, 1)

   Connect the points in order starting and ending at (1, 1). Add an eye to complete your drawing.

Use **Student Activity Sheet 4** for problems 8–12.

8. a. Plot the following points on your coordinate system. Remember that the first coordinate of the pair names a position going right or left in the horizontal direction, and the second coordinate names a position going up or down in the vertical direction.

   (1,1), (5,1), (6,2), (7,2), (7,1), (8,1), (9,2), (9,4), (7,4), (6,5), (5,5), (1,3), (0,3), (1,1)

   b. Connect the points in the order they are shown in 8a. What is the result?

For problems 9 and 10, predict what you think will happen to the drawing you made for problem 8. Check your prediction by making a new drawing. For each problem, start with the drawing you made for problem 8.

9. Add −10 to the first coordinate of each point. What happens?

10. Add 2 to the first coordinate and add −5 to the second coordinate of each point. What happens?

11. What should you do to the coordinates if you want to move the drawing up three units and to the right five units?

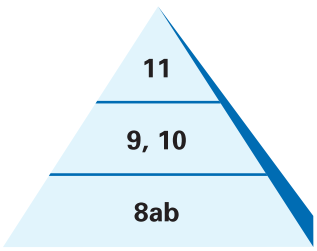
12. What should you do to the coordinates if you want the drawing to become twice as large?

Notes

7 and 8 If students are having trouble plotting the points, provide them with extra practice by using Cartesian cartoons to graph. These are lists of coordinates that when connected make a picture.

12 Their first impulse is to double both coordinates. Have them try out their theory and then discuss why it didn't work.

Assessment Pyramid



Understand the similarity of using integers in algebraic and geometric contexts. Plot ordered pairs.

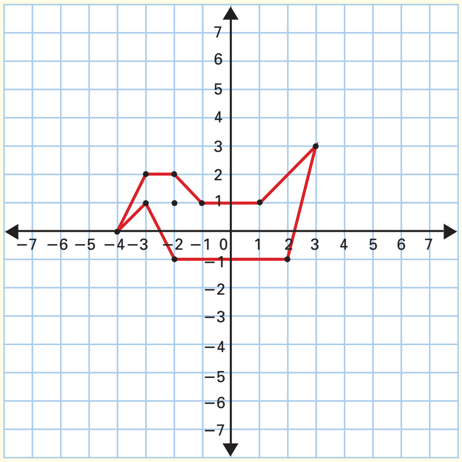
Reaching All Learners

Hands-On Learning

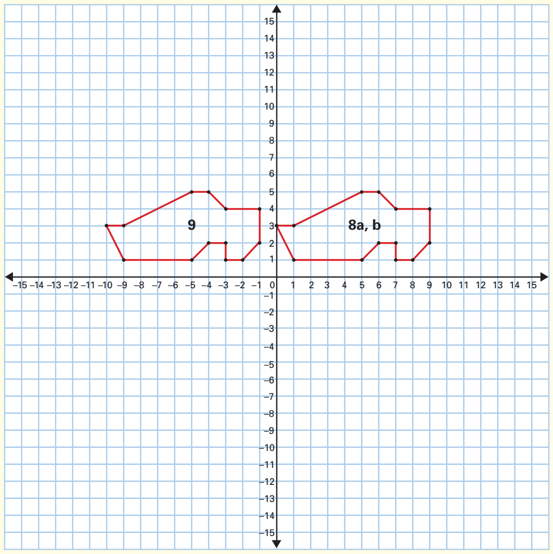
After discussing problem 12, have students form a human 2 × 2 box (four students standing in two rows and two columns). Ask students how they would change the dimensions of the "box" to double the number of students. If they choose to double both directions, add the appropriate number of students to show how their guess was incorrect.

Solutions and Samples

7 a. and **b.**

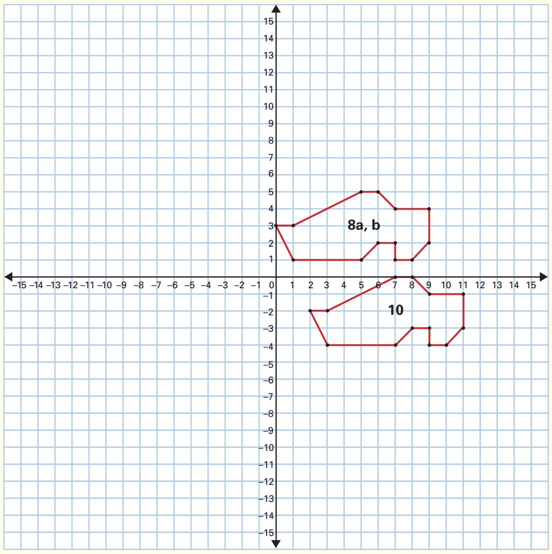


8 a., b. and **9.**



9. (See above drawing.) The wooden shoe moves ten units to the left.

10. The wooden shoe moves two units to the right and down five units.



Hints and Comments

Materials

Student Activity Sheet 4 (one per student); ruler or straightedge (one per student)

Overview

Students plot points on a coordinate system. They predict what will happen to a figure when the coordinates are changed, and then verify the actual result by graphing each change.

About the Mathematics

When you operate on the coordinates of a figure, several transformations are possible:

- the figure, when a number is added or subtracted from the coordinates: (1) when only the first coordinate changes, it is a shift in the left/right direction; and (2) when only the second coordinate changes, it is a shift in the vertical direction;
- enlarging or reducing the figure, when the coordinates are multiplied or divided by a number: (1) when the first coordinate changes, it is an enlargement or a reduction in the left/right direction; and (2) when the second coordinate changes it is an enlargement or a reduction in the vertical direction.

Comments About the Solutions

7. Students do not need to label the axes with *x* and *y*. At this moment, the formalization is not necessary. Both the letter O and (0, 0) can be used for the origin.

9–12.

Some students may make new, translated figures. Others may use adjustments to an existing figure. It may be worthwhile to have students share their strategies.

12. Discuss with students what the meaning is of *twice as large*. This might be a good problem for students to practice finding area if they have done the unit *Reallotment*.

11. Add 5 to the first coordinate and add 3 to the second coordinate of each pair.

12. The answer depends on what is meant by "twice as large." If you multiply each coordinate by two, all measurements will become twice as long. However, the area of the drawing would become four times as big! To get an area that was twice as big, you would have to double only the first or the second coordinate from each pair.

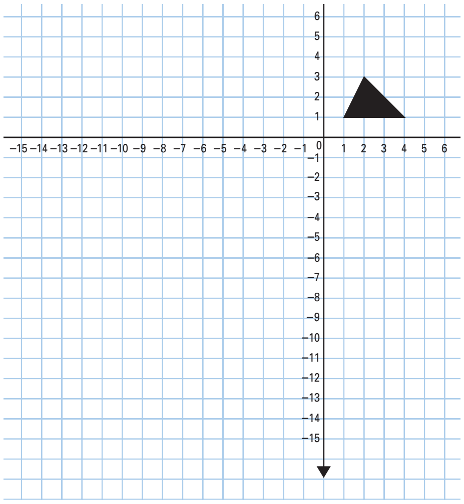
E Operations and Coordinates

Notes

13b This appears to be only 8 times bigger when students re-draw it. You need to emphasize that it is 9 times bigger.

14 You can ask students what to do to the triangle to make the statements correct. You may want to have a class discussion of the reasoning each of the four classmates followed.

14 and 15 Ask students to try out their hypotheses to verify if they are correct.



13. On a new copy of **Student Activity Sheet 4,** draw the triangle shown on the graph. Only part of the graph is shown on the left.
    a. What are the coordinates of each **vertex**?
    b. Multiply all of the coordinates by −3. What are the new coordinates of the vertices?
    c. Draw the new figure in the same coordinate system. Describe how the new figure is related to the original one.

Carrie wonders what would happen to the figure she made in problem 13 if she multiplied the coordinates by −3 a second time. This is what some of her classmates think.

John says, "It would be upside down and three times as big."
Mauri says, "I guess it would be nine times as big."
Taye says, "I think it would become the first figure."
Emily says, "The coordinates of the top point would be (9, 8)."

14. a. **Reflect** Comment on the thinking of each of Carrie's four classmates.
    b. Write down everything you know about what will happen to the figure.

15. a. What would happen to the triangle you drew for problem 13a if you multiplied only the first coordinate by −3 and kept the second coordinate the same?
    b. What would happen to the triangle you drew for problem 13a if you kept the first coordinate the same and multiplied only the second coordinate by −3?

Reaching All Learners

Intervention

You may want to add this chart to the bottom of **Student Activity Sheet 4** to help students organize their work:

Vertex	13a Original Coordinates	13b Multiply All Coordinates by (–3)
A		
B		
C		

Vocabulary Building

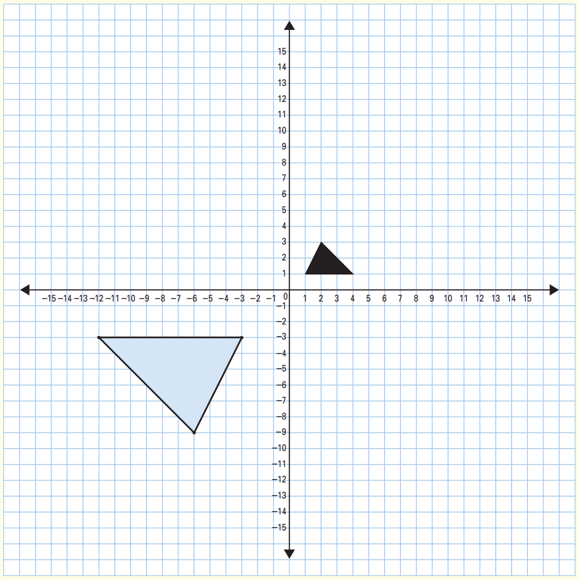
Student may need to be reminded that *vertex* is the point where the sides of the triangle meet.

Solutions and Samples

13. a. (1,1), (4,1), (2,3)

b. (−3,−3), (−12,−3), (−6,−9)

c.



Answers will vary:

- The new triangle has 9 times the area of the original triangle.
- Each side is 3 times longer than the corresponding side of the original triangle.

14. a. Answers and explanations will vary.

Mauri's answer is correct about the area of the new figure since multiplying every coordinate by three will increase the area by 3^2. However multiplying by −3 twice is like multiplying by 9 so the area will be $9^2 = 81$ times bigger.

John's answer is incorrect; the lengths are three times as big, but the area is not. If the length of each side is multiplied by three, the area is 3 × 3, or nine times larger. The triangle is in the same position as the original drawing.

Taye's answer is incorrect. To get back to the original figure, one has to multiply by $-\frac{1}{3}$.

Emily's answer is also incorrect. The coordinates of the top point would be (18, 27).

Sample student work:

- None of the students are right.
- Mauri is wrong because if you multiply −3 again, it will be right side up, and if you times again, it will be 81 times bigger.
- Taye: If you multiply by a whole number, it won't shrink, it will get big.
- If you multiply it by −3, it will flip. If you do that again, it will flip again, getting bigger.

Hints and Comments

Materials

Student Activity Sheet 4 or graph paper (one sheet per student);
ruler or straightedge (one per student)

Overview

Students investigate the effect of multiplication on the coordinates of given figures.

About the Mathematics

When the vertical coordinates of a figure are multiplied by −1 and the horizontal coordinates remain the same, the figure flips upside down with the horizontal axis as the line of reflection. When the horizontal coordinates are multiplied by −1 and the vertical coordinates remain the same, a figure flips left/right with the vertical axis as the line of reflection.

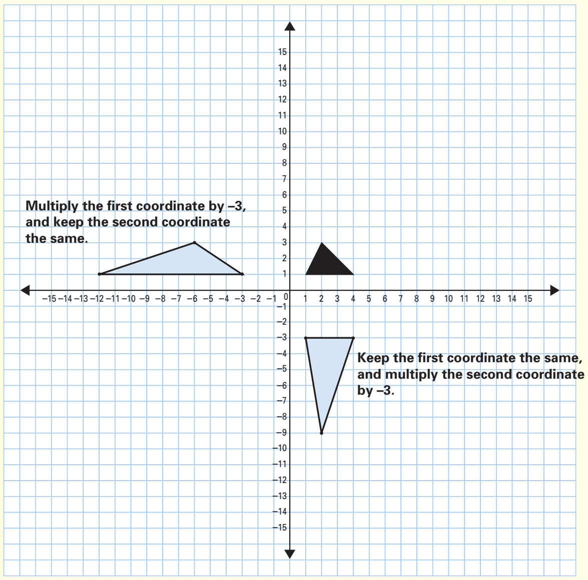
Many of the changes of the figures in this section can be broken up into smaller steps, for example, first a flip, then an enlargement, and then a shift.

b. Answers will vary. Sample response:

The coordinates (9,9) of the vertices of the new figure will be (36, 9) and (18, 27). The figure will have an area 81 times the area of the original figure.

15. a. The triangle moves to the left and 'stretches' in the horizontal direction.

b. The triangle moves down and its position is reversed. It 'stretches' in the vertical direction.



E Operations and Coordinates

Notes

16 Have students try their hypothesis by drawing it. Any fraction between 1 and −1 will work, not just $\frac{1}{2}$.

After students have finished problem 16, you may choose to have a class discussion about enlarging and shrinking figures. This could include some discussion about the relationship between the operations of multiplication and division.

17–19 Have students make a chart of the original coordinates and the new coordinates and figure out what they would do to get from the original to the new.

Gil, Lashonda, and Greg are discussing how they might shrink a triangle.

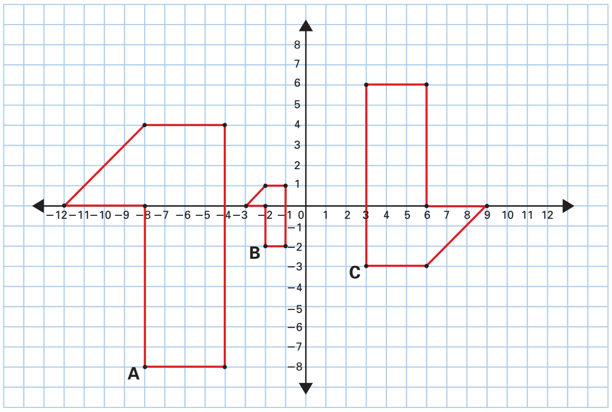
Gil says, "You could multiply the coordinates by −2."

Lashonda says, "That is not right. You would have to multiply the coordinates by $\frac{1}{2}$."

Greg says, "Why not multiply by $-\frac{1}{2}$?"

16. Which of these statements do you think is/are correct?

Three figures have been drawn in the coordinate system below. Use a new copy of **Student Activity Sheet 4** for problems 17–19.



17. **a.** Choose one of the figures A, B, or C. Describe how you can get the other two figures from the one you chose by multiplying or dividing the coordinates of all of the points of that figure.

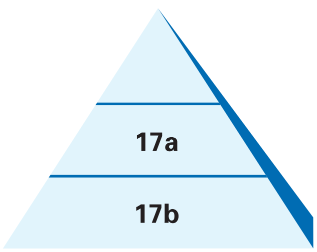
b. Check your answer and show how you checked it.

For problems 18 and 19, you will multiply a figure by a number. This means that you will multiply the coordinates of all of the points by that number.

18. Start with figure B and multiply figure B by −1. Draw the new figure on another copy of **Student Activity Sheet 4** and name it D.

19. Multiply figure D by −1. What do you notice? What does this tell you about −1 × −1?

Assessment Pyramid



Explore transformations of geometric figures in a coordinate system.

Reaching All Learners

Intervention

Many students automatically associate multiplying with getting bigger. You may want to discuss some examples, like 10 multiplied by $\frac{1}{2}$, is 5, which is smaller. You may also want to discuss how multiplying by a fraction like $\frac{1}{2}$ gives the same result as dividing by 2.

Writing Opportunity

You may have students write their comments (problem 14b) in their notebooks as well as their answer to problem 15. This will give you some insight into their progress and their ability to reason mathematically.

Solutions and Samples

16. Answers will vary. Both Lashonda's and Greg's suggestions would shrink a triangle (or any other figure).

Greg's suggestion will also flip it.

Sample student work:

- Lashonda is right; you can't multiply by −2; it would just be bigger. You can multiply by $\frac{1}{2}$ and it will become smaller. And if you multiply the coordinates by $-\frac{1}{2}$, it still shrinks. It shrinks because it's a fraction (or a decimal) and it doesn't matter if it is positive or negative.
- Gil is wrong because it will only get bigger. Lashonda is right because you have to multiply a number smaller than one. Greg is also correct because it doesn't matter if it's negative or positive.

17. a. Starting from A:

Get to B by multiplying by $\frac{1}{4}$.

Get to C by multiplying by $-\frac{3}{4}$.

Starting from B:

Get to A by multiplying by 4.

Get to C by multiplying by −3.

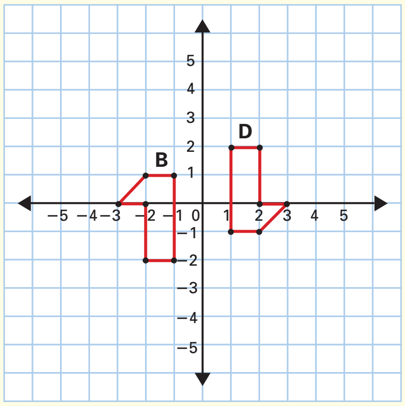
Starting from C:

Get to A by multiplying by $-\frac{4}{3}$.

Get to B by multiplying by $-\frac{1}{3}$.

b. Responses will vary. Some students may check by showing the calculations for specific points. For example, to get from B to A, the point (−3, 0) becomes (−12, 0), so the coordinates were multiplied by 4.

18.



19. You get figure B back. This suggests that −1 × −1 = 1.

Hints and Comments

Materials

Student Activity Sheet 4 or graph paper (two sheets per student);
ruler or straightedge (one per student)

Overview

Students continue to investigate enlarging and reducing (or shrinking) figures.

About the Mathematics

Multiplication and division are very much related. For example, multiplying by $\frac{1}{3}$ is the same as dividing by three. Shrinking or reducing a figure can be done by dividing the coordinates or by multiplying the coordinates by a fraction between −1 and +1.

To undo an operation, the inverse operation can be performed. For example, a multiplication by −3 can be undone by a division by −3 or by a multiplication by $-\frac{1}{3}$.

Comments About the Solutions

18. and **19.**

In these problems, the notion of undoing an operation is investigated. The inverse operation of multiplying by −1 is dividing by −1, and division by −1 is the same as multiplication by −1.

Operations and Coordinates

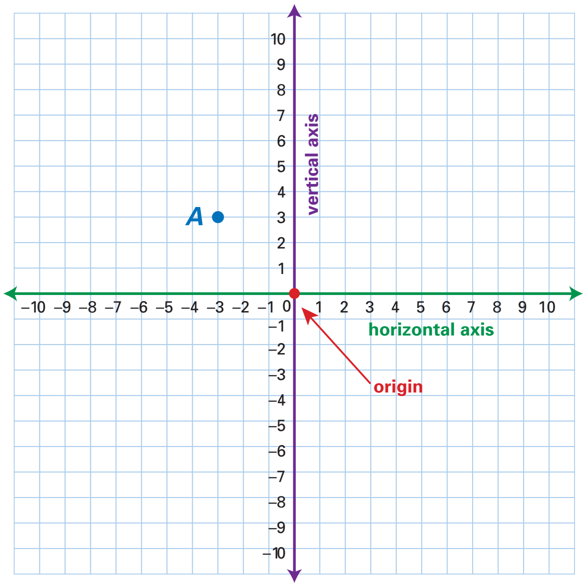
Notes

Read the Summary out loud. Discuss areas in the section where the information was applied.

Operations and Coordinates

Summary

To locate points on a grid, you can use a coordinate system. The origin is the point at the intersection of the horizontal axis and the vertical axis, which are like perpendicular number lines. Each point can be located by using two coordinates that tell where the point is located relative to the origin. Positive directions are up and to the right; negative directions are down and to the left.



You can change the position of a figure drawn in a coordinate system by adding a number to or subtracting a number from each coordinate, or by multiplying or dividing the coordinates by some number. You need to remember the rules for these operations in order to do this.

Reaching All Learners

Vocabulary Building

This Summary section contains much of the terminology that was discussed in this section. Students should be able to describe words like *horizontal, vertical, coordinate,* and *origin* in their own words.

Hints and Comments

Overview

Students review the mathematical concepts involved with the transformations of figures in a coordinate system. Students use the Check Your Work problems as self-assessment. The answers to these problems are also provided on Student Book pages 62 and 63.

About the Mathematics

The mathematics in this section is about adding, subtracting, multiplying, and dividing positive and negative numbers, all applied in a geometric context. However, the rules discovered in this section about operations with positive and negative numbers are also true for operations in a non-geometric context.

Planning

After students complete Section E, you may assign as homework appropriate activities from the Additional Practice section, located on Student Book page 57.

E Operations and Coordinates

Notes

Make sure students share their explanations for problems 2 and 3.

Check Your Work

1. a. Draw your own coordinate system on grid paper by making vertical and horizontal axes 14 units long. Put a number scale on each axis, starting with 0 at the origin. Plot the following points and connect the points with line segments in this order.

   (−5, 0), $(-2\frac{1}{2}, 1)$, (−1, 1), (−1, 3), (1, 6), (1, 3), (2, 1), (7, 1), (5, 0)

   b. Use the horizontal axis as a mirror to draw the other half of the picture of a bird in flight.

   c. Write the coordinates of the points you drew for problem 1c.

2. Do you think the following statement is always true? Give an explanation for your answer.

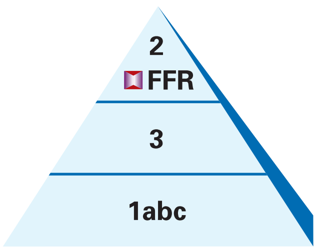
   "If you multiply or divide the coordinates of a figure by a number, the size will always change."

3. Suppose a figure called A is multiplied by +2 and called figure B. Then figure B is multiplied by −1 and called figure C, and figure C is multiplied by $+\frac{1}{2}$ and called figure D. How can you get figure D directly from figure A?

For Further Reflection

How can you make a figure smaller by multiplying?

Assessment Pyramid



Assesses Section E Goals

Reaching All Learners

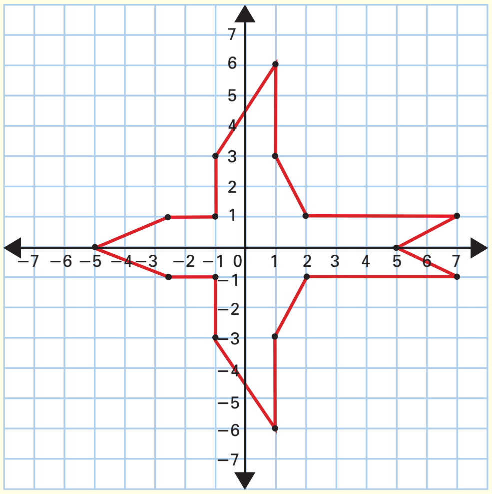
Writing Opportunity

This would be a good time to summarize students' collective observations about how shapes in the coordinate plane change when using various operations and values (e.g., less than one, negative, positive) with the coordinates. These observations should be recorded in students' notebooks or journals.

Solutions and Samples

Answers to Check Your Work

1. a. and **b.**



c. $(-2\frac{1}{2}, -1)$ $(-1, -1)$ $(-1, -3)$ $(1, -6)$ $(1, -3)$ $(2, -1)$ $(7, -1)$

2. Compare your answer with a classmate. No, the statement is not always true. Sample response:

The size of the figure will always change, unless you multiple all coordinates by 1 or −1.

3. You can get figure D by multiplying the coordinates of figure A by −1. If you did not find the answer to this question, make up an example for figure A. You could draw figure A as a rectangle and then draw figures B, C, and D following the instructions.

Doing each multiplication separately has the same results as multiplying all the numbers and then making that change to the figure.

$$2 \times -1 \times \frac{1}{2} = -1$$

For Further Reflection

If you have a figure on a coordinate plane, you can make the figure smaller either by dividing each of the coordinates by a number greater than 1 or by multiplying the coordinates by a number between 0 and 1. You can also divide or multiply by negative numbers to make a figure smaller, but using negative numbers will do more than just shrink the figure.

Hints and Comments

Materials

ruler or straightedge (one per student)

Overview

Students use the Check Your Work problems as self-assessment. The answers to these problems are also provided on Student Book pages 62 and 63.

Additional Practice

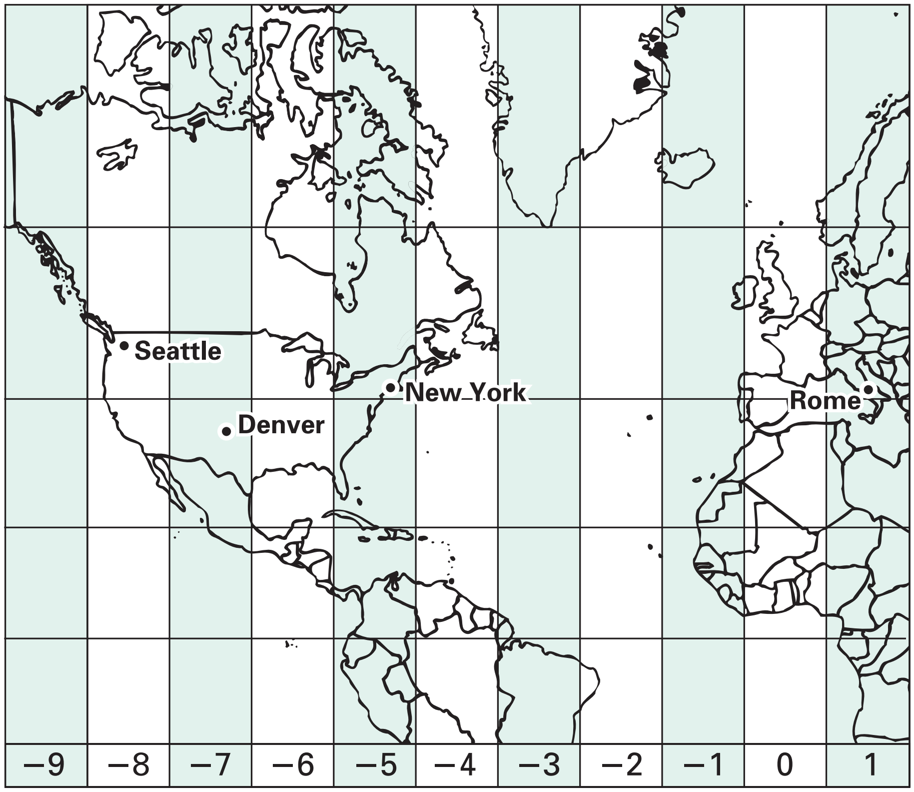
Section A Positive and Negative

Sharon Taylor is a salesperson for a toy manufacturing company. To sell her company's toys to different stores, she must travel quite often. Sharon is flying from New York to Seattle for a meeting with a retail chain.

1. Sharon's plane left New York at 3:00 P.M., and it is a six-hour flight from New York to Seattle. What time will she arrive in Seattle?

After her trip to Seattle, Sharon returns to New York. She wants to schedule a conference call by telephone with an Italian toy retailer in Rome. Sharon works between 9:00 A.M. and 5:00 P.M. each day. The Italian retailers also work from 9:00 A.M. to 5:00 P.M.

2. When can Sharon schedule a conference call?



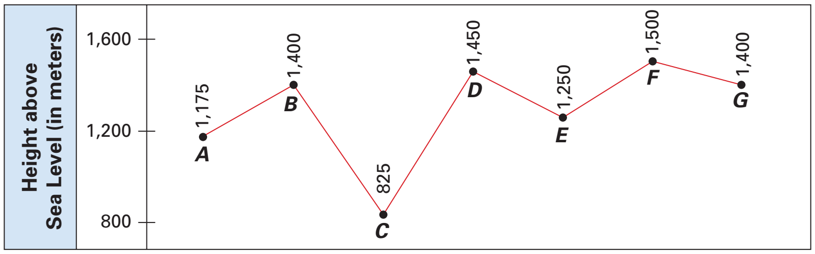
Section A. Positive and Negative

1. She will arrive at 6:00 P.M. There is a three-hour time difference, and it is a six-hour flight. She will gain three hours, so she will arrive at 6:00 P.M.

2. The hours 9 A.M. to 5 P.M. in New York correspond to 9:00 to 17:00. In Rome, it is 15:00 to 23:00. So if Sharon wants to talk between 9:00 and 17:00 New York time, she should call between 15:00 and 17:00 Rome time. This corresponds to the interval from 9:00 A.M. to 11:00 A.M. New York time.

   Rome is in time zone 1, while New York is in time zone −5, so the difference is six hours.

A group is planning a hiking trip from point *A* to point *G* on the trail mapped below.



Someone in the group says, "Wow, we are going to be only a few hundred meters higher when we end. That does not sound like a difficult hike." As the hiking guide, you decide it would be helpful to explain how much climbing they will have to do.

3. a. How many meters is the difference in height between point *A* and point *B*? How many meters does the group descend between points *B* and *C*?

b. Complete the table, which shows how many meters the hikers will have to ascend and descend. Use + for going up and – for going down.

	Height (in m)	Up	Down	
A	1,175			
B	1,400	+225		
C	825		–575	
D	1,450			
E	1,250			
F	1,500			
G	1,400			
Total		+	–	=

c. What are the totals in the columns "Up" and "Down"? What does this mean for the difference in height between points *A* and *G*?

d. Use the table you made for problem 3b to decide if this is a difficult hike. Give mathematical reasons to support your answer. If you want to, you may use the general rule: There are about 3 ft in 1 m.

Section A. Positive and Negative (continued)

3. a. From point *A* to point *B*, you go up 225 m.
1,400 − 1,175 = 225

From point B to point C, you go down 575 m.
1,400 − 825 = 575 m

b. and **c.**

	Height (in m)	Up	Down	
A	1,175			
B	1,400	+225		
C	825		−575	
D	1,450	+625		
E	1,250		−200	
F	1,500	+250		
G	1,400		−100	
Total		+1,100	−875	= +225

The difference in height between points *A* and *G* is 225 m.
+1,100 − 875 = +225

d. Sample student answers:

- If you are not an experienced hiker, this trail MIGHT BE rather difficult because you have to go up 1,100 meters or about 3,300 feet and you do not know how many days you have for doing this.
- You have to go up a total of 1,100 m, which is quite a lot even for one day. Moreover, you descend a total of 875 m.

Additional Practice

Section B Walking Along the Number Line

Draw a number line exactly 14 centimeters (cm) long. Use your ruler. Put −7 on the left end of the number line, +7 at the right end, and 0 in the middle.

1. **a.** Use an arrow to indicate −4 on your number line.

   b. Use an arrow to indicate 2.8 on your number line.

   c. What is the distance between −4 and 2.8?

2. The lowest point in Washington, D.C., is +0.3 m, and the lowest point in Louisiana is −2.4 m. What is the difference in height between these two points?

3. **a.** Use a short notation to write "negative seven is less than positive two."

   b. Make true statements using $<$ or $>$ or $=$.

   +75 ____ +57 −100 ____ +10 $5\frac{3}{4}$ ____ 5.75

   − 3 ____ +3 −100 ____ −1000 $-2\frac{1}{2}$ ____ $-2\frac{2}{5}$

4. The temperature in Bergen, Norway, was − 8 degrees Celsius overnight. During the daytime, the temperature went up to a maximum of +4 degrees Celsius. How much did the temperature rise?

5. Complete the following.

   a. −25 ADD 17 → ____

   b. 25 ADD −17 → ____

   c. 25 SUBTRACT 25 → ____

   d. 25 SUBTRACT −25 → ____

   e. −25 SUBTRACT ____ → 50

Section C Calculating with Positive and Negative Numbers

1. Fill in the blanks.

   a. Adding 8 gives the same result as subtracting ______.

   b. Subtracting 10 gives the same result as adding ______.

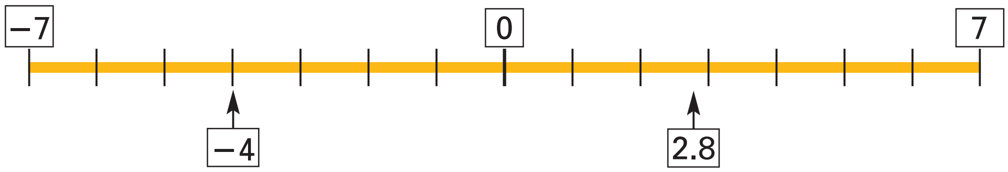
   c. 15 − (−3) = ____

   d. 15 + ____ = 18

Section B. Walking Along the Number Line

1. a. and **b.**

Note that the number line must be drawn with the aid of a ruler and the distance between the numbers on the line must be equal, one centimeter each.



c. The distance between −4 and 2.8 is 6.8.

2. The difference in height is 2.7 m.

3. a. $-7 < +2$

b. $+75 > +57$ $-100 < +10$ $5\frac{3}{4} = 5.75$

$-3 < +3$ $-100 > -1000$ $-2\frac{1}{2} < -2\frac{2}{5}$

4. The temperature rose 12°C.

5. The lines completed:

a. −25 ADD 17 → −8

b. 25 ADD −17 → 8

c. 25 SUBTRACT 25 → 0

d. 25 SUBTRACT −25 → 50

e. −25 SUBTRACT −75 → 50

Section C. Calculating with Positive and Negative Numbers

1. a. Adding 8 gives the same result as subtracting −8.

b. Subtracting 10 gives the same result as adding −10.

c. $15 - (-3) = 18$

d. $15 + 3 = 18$

Additional Practice

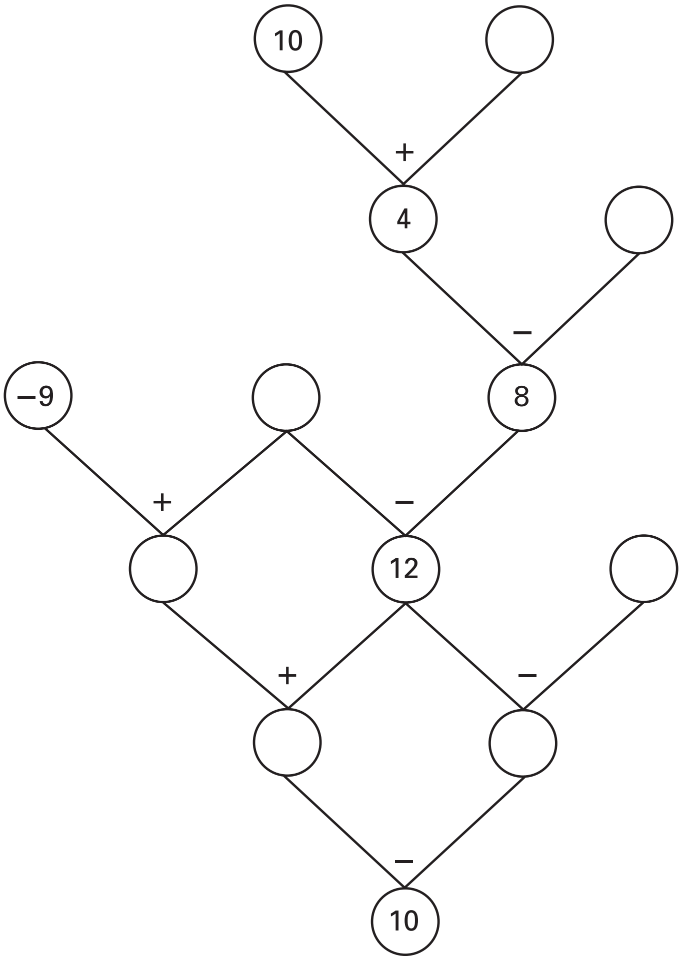
Here is a list of high temperatures in degrees Celsius for one week at a ski lift station.

Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat
High Temperature (°C)	−7	−5	−5	−3	−5	−7	−6

2. Find the mean high temperature for that week at the station. Show your work.

3. The following tree uses addition and subtraction. Copy and fill in the tree going from left to right.

If the sign is negative (−), you have to subtract the number on the right from the number on the left.



Section C. Calculating with Positive and Negative Numbers (continued)

2. The mean high temperature is about -5.4, or $-5\frac{3}{7}$°C.

Sample student work:

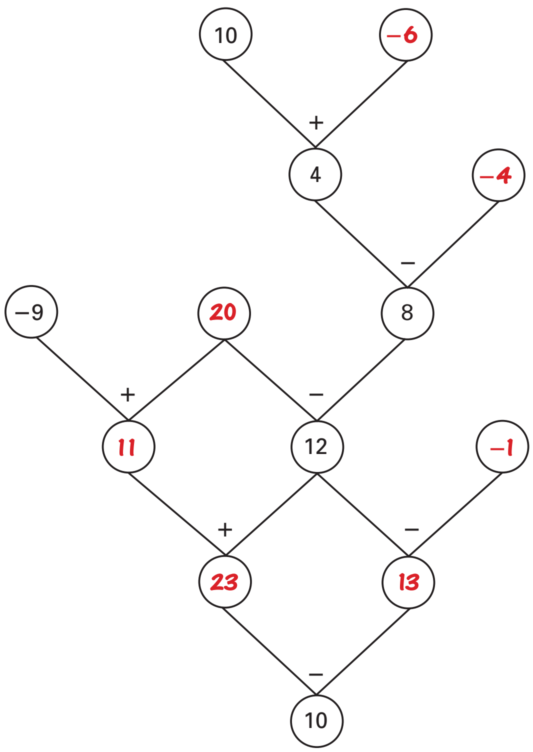
$2 \times -7 = -14$

$3 \times -5 = -15$

$-14 + -15 + -3 + -6 = -38$

$-38 \div 7 = -5\frac{3}{7}$, or about -5.4

3.



Additional Practice

Section D Adding and Multiplying

1. Are the following statements always true? If not, show an example for which the statement is not true.

a. A positive number multiplied by a positive number gives a positive number.

b. A positive number added to a negative number gives a positive number.

c. A negative number multiplied by a negative number gives a positive number.

The students in Ms. Makuluni's class have measured their pulse rate for half a minute while sitting at their desks. Here are the results.

34	35	35	36	35	36
33	37	32	34	36	34
30	37	34	38	33	35
31	35	36	33	38	37

2. a. Make a list of positive and negative differences from 35.

b. Is the mean pulse rate, measured for half a minute, more or less than 35? Show your work.

3. Complete the calculations in both tables.

Table 1

+	−8	−5	−2	1	4.5
6					
2		−3			
−2					
−6					
−10					

Table 2

×	−8	−5	−2	1	4.5
6					
2					
−2					
−6		30			
−10					

4. Write five different calculations using positive and negative numbers.

Fractions and decimal numbers are also allowed.
You may use addition, subtraction, and multiplication.
Don't make your problems too hard! You must include a list with correct answers for your calculations.

Section D. Adding and Multiplying

1. a. Always true.

b. Not always true; for example,
$-16 + 8 = -8$, which is not a positive number.

c. always true

2. a.

−1	0	0	+1	0	+1
−2	+2	−3	−1	+1	−1
−5	+2	−1	+3	−2	0
−4	0	+1	−2	+3	+2

b. The total of the differences is −6; the mean is less than 35.

3. The tables completed:

Table 1

+	−8	−5	−2	1	4.5
6	−2	1	4	7	10.5
2	−6	−3	0	3	6.5
−2	−10	−7	−4	−1	2.5
−6	−14	−11	−8	−5	−1.5
−10	−18	−15	−12	−9	−5.5

Table 2

×	−8	−5	−2	1	4.5
6	−48	−30	−12	6	27
2	−16	−10	−4	2	9
−2	16	10	4	−2	−9
−6	48	30	12	−6	−27
−10	80	50	20	−10	−45

4. Answers will vary. The answers given by the students give an indication of their level of understanding of operations with positive and negative numbers.

Section E Operations and Coordinates

Use graph paper for problems 1–5.

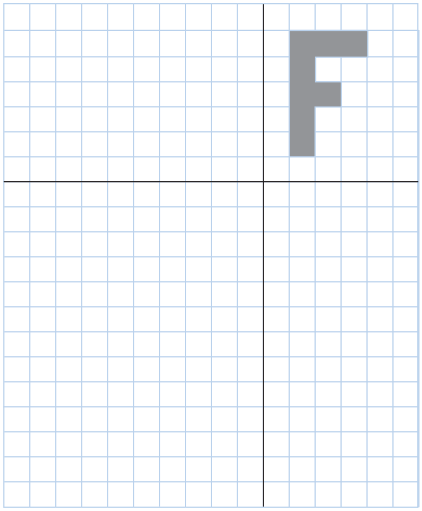
1. a. Draw a coordinate system. Put number scales on it and mark 0 for the origin.

   b. Plot these points on your coordinate system: A (−1, −2), B (3, −2), C (4, 1), and D (0, 1).

   c. Connect the points in alphabetical order.

2. What are the new coordinates of your figure if you move it three spaces to the left and also three spaces up? Write the coordinates and draw the new figure in the same coordinate system you made for problem 1.

3. On **Student Activity Sheet 5,** you see a drawing of a letter F in a coordinate system.



   a. Multiply each first coordinate of the letter F by −2 and keep each second coordinate the same. Draw the new figure in the same coordinate system and mark it with an A. Use a ruler.

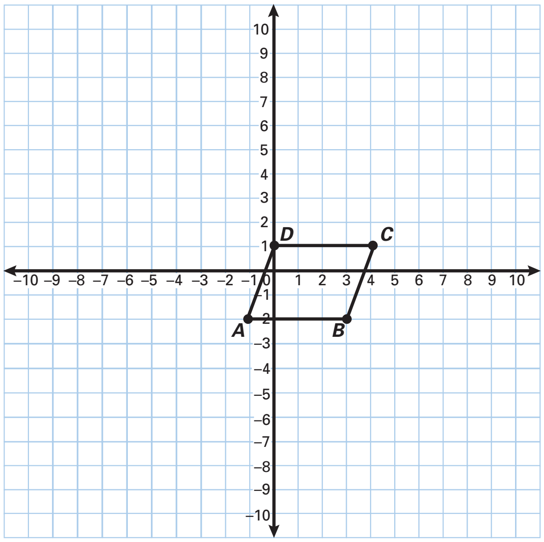
   b. Keep each first coordinate of the original letter F the same and now multiply each second coordinate by −2. Draw the new figure in the coordinate system and mark it with a B.

   c. How did the shape of the original letter F change in parts **a** and **b**?

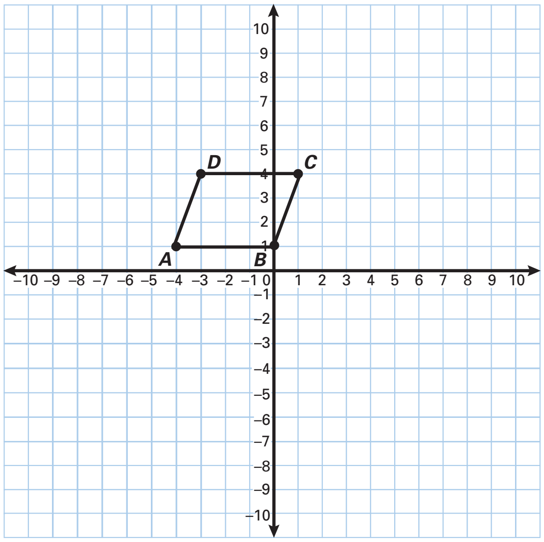
   d. What happens if you multiply both coordinates of the original letter F by −2? Make a drawing to support your reasoning.

Section E. Operations and Coordinates

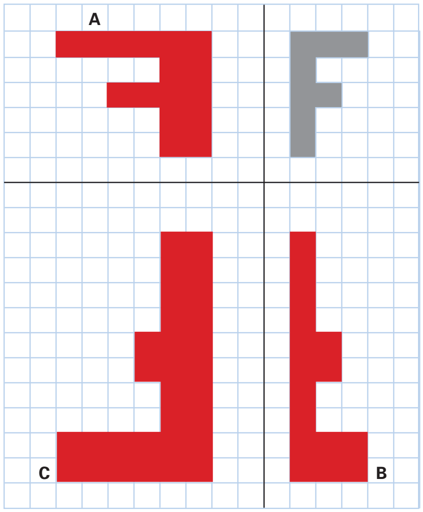
1.



2. $A(-4, 1)$ $B(0, 1)$ $C(1, 4)$ $D(-3, 4)$



3.



c. In A, all of the horizontal segments are doubled. The direction of the letter reversed rather like a horizontally “stretched” mirror image across the *y*-axis.

For B, all of the vertical segments are doubled and the letter is reflected across the horizontal axis. It has been “stretched” vertically.

d. In C, the letter looks twice as big because it was stretched both horizontally and vertically by 2. It has been reflected over each of the axes so it is upside down and facing the opposite direction from the original F.

Assessment Overview

Unit assessments in *Mathematics in Context* include two quizzes and a Unit Test. Quiz 1 is to be used anytime after students have completed Section B. Quiz 2 can be used after students have completed Section D. The Unit Test addresses most of the major goals of the unit. You can evaluate student responses to these assessments to determine what each student knows about the content goals addressed in this unit.

Pacing

Each quiz is designed to take approximately 25 minutes to complete. The end of unit test was designed to be completed during a 45-minute class period. For more information on how to use these assessments, see the Planning Assessment section on the next page.

Goals	Assessment Opportunities		Problem Levels
• Describe patterns using positive and negative numbers.	Quiz 2 Test	Problems 3abcd Problems 1ab	I
• Compare and order positive and negative numbers .	Quiz 1 Quiz 2 Test	Problems 2abcde, 3abc Problems 3abcd Problems 2abcd	
• Perform operations with positive and negative numbers.	Quiz 1 Quiz 2 Test	Problems 1ab, 3abc, 4 Problems 2abcd, 3abc, 4 Problems 3abcdef, 4abcd, 5	
• Name and plot ordered pairs on a coordinate plane.	Test	Problems 6abcd, 7ab	
• Recognize and use the property of opposites (canceling out positive and negative numbers).	Quiz 1 Test	Problems 1ab, 4 Problem 3f	II
• Understand the similarity of using intergers in algebraic and in geometric contexts.	Test	Problems 1ab, 7c	
• Explore transformations of geometric figures on a coordinate system.	Test	Problems 7ab	
• Reason about and predict transformations of geometric figures in a coordinate system.	Test	Problem 7c	III
• Generalize rules for operating with positive and negative numbers.	Test	Problem 4e	

About the Mathematics

These assessment activities assess the majority of the goals for *Operations*. Refer to the Goals and Assessment Opportunities section on the previous page for information regarding the goals that are assessed in each problem. Some of the problems that involve multiple skills and processes address more than one unit goal. To assess students' ability to engage in non-routine problem solving (a Level III goal in the assessment pyramid), some problems assess students' ability to use their skills and conceptual knowledge in new situations. For example, in the transformation of the triangle problem on the Unit Test (problem 7), students must use their knowledge of inverse operations and the effect of operations on shapes in the coordinate plane to propose a method for solving a new problem.

Planning Assessment

These assessments are designed for individual assessment, however some problems can be done in pairs or small groups. It is important that students work individually if you want to evaluate each student's understanding and abilities.

Make sure you allow enough time for students to complete the problems. If students need more than one class session to complete the problems, it is suggested that they finish during the next mathematics class or you may assign select problems as a take-home activity. Students should be free to solve the problems their own way. Most of these problems on these assessments are designed to be calculator-free. However student use of calculators is left to the teacher's discretion.

If individual students have difficulties with any particular problems, you may give the student the option of making a second attempt after providing him or her a hint. You may also decide to use one of the optional problems or Extension activities not previously done in class as additional assessments for students who need additional help.

Scoring

Solution and scoring guides are included for each quiz and the unit test. The method of scoring depends on the types of questions on each assessment. A holistic scoring approach could also be used to evaluate an entire quiz.

Several problems require students to explain their reasoning or justify their answers. For these questions, the reasoning used by students in solving the problems, as well as the correctness of the answers, should be considered in your scoring and grading scheme.

Student progress toward goals of the unit should be considered when reviewing student work. Descriptive statements and specific feedback are often more informative to students than a total score or grade. You might choose to record descriptive statements of select aspects of student work as evidence of student progress toward specific goals of the unit that you have identified as essential.

Name ____________________ Date ____________

Operations Quiz 1

Use additional paper as needed.

1. Ernie wants to buy a pair of inline skates. He tries to save money, and sometimes he can do some odd jobs to earn some extra money. On May 1, he had $24.

This is how Ernie keeps track of the changes in his savings.

24 $\xrightarrow{+18}$ ____ $\xrightarrow{-5}$ ____ $\xrightarrow{?}$ ____ $\xrightarrow{+35}$ ____ $\xrightarrow{+18}$ 85

Ernie cannot remember what happened one day, so there is a question mark.

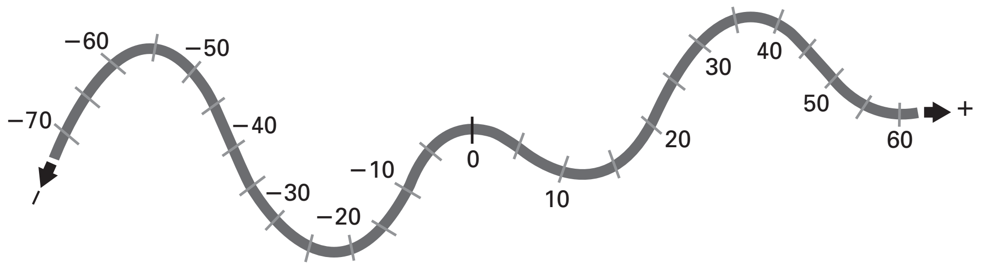
a. Did Ernie spend money or earn money on the day represented by the question mark?

b. What was the amount?

2. In the table below, mark the statements as true or false.

	True	False
a. $0 > -5$		
b. $-18 < -20$		
c. $-35 < -34$		
d. $999 > -1000$		
e. $-2.04 > -1.98$		

3. Look at the curved number line.



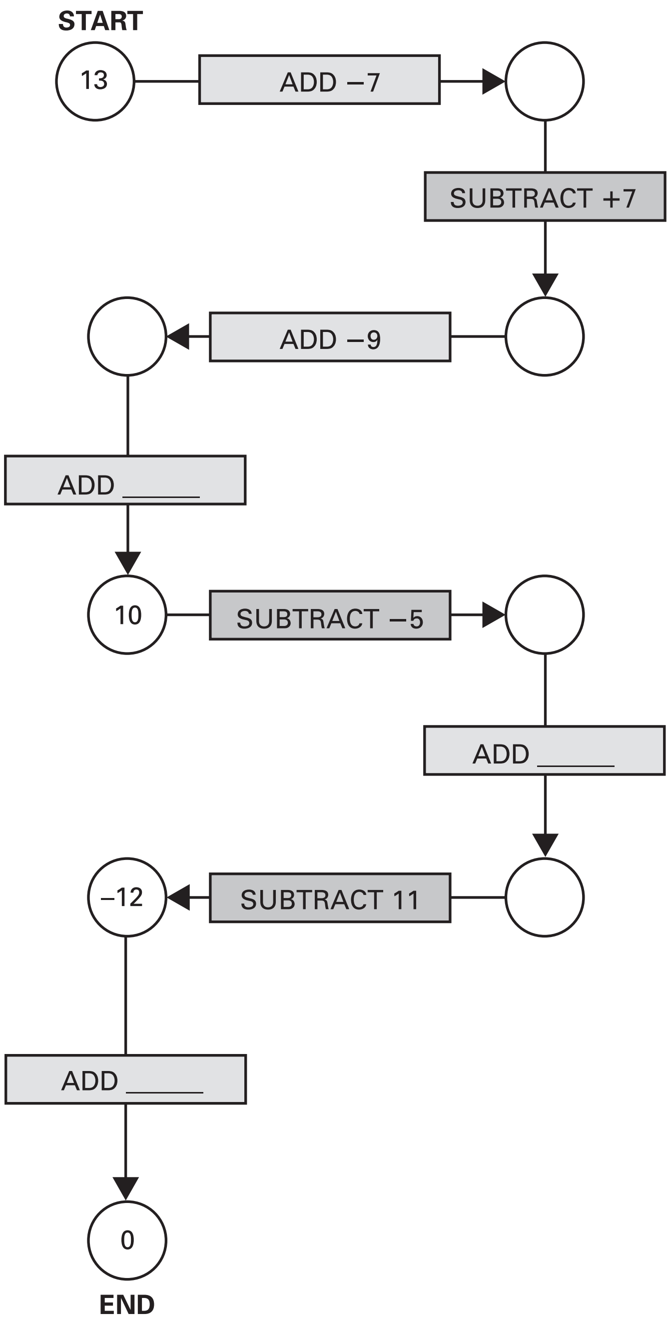
Using the number line, find the difference between the numbers.

a. 60 and −70 Answer: ________

b. 55 and −45 Answer: ________

c. −15 and −40 Answer: ________

4. Complete the following series of instructions.



Name ______________________ Date ______________

Operations Quiz 2

Use additional paper as needed.

On windy days in winter, it often feels much colder than the temperature on the thermometer indicates. This is called *wind chill.* One day, the weather station announces:

Tomorrow the temperature will be around −5° F at noon. But due to the wind chill, it will feel as if it is eight degrees colder.

1. What temperature will it feel like at noon tomorrow?

2. Fill in the missing number to make a true statement.

a. 25 + _____ = 56

b. 25 − _____ = 56

c. 25 + _____ = − 75

d. _____ − 25 = − 75

3. Make each statement true using <, =, or >.

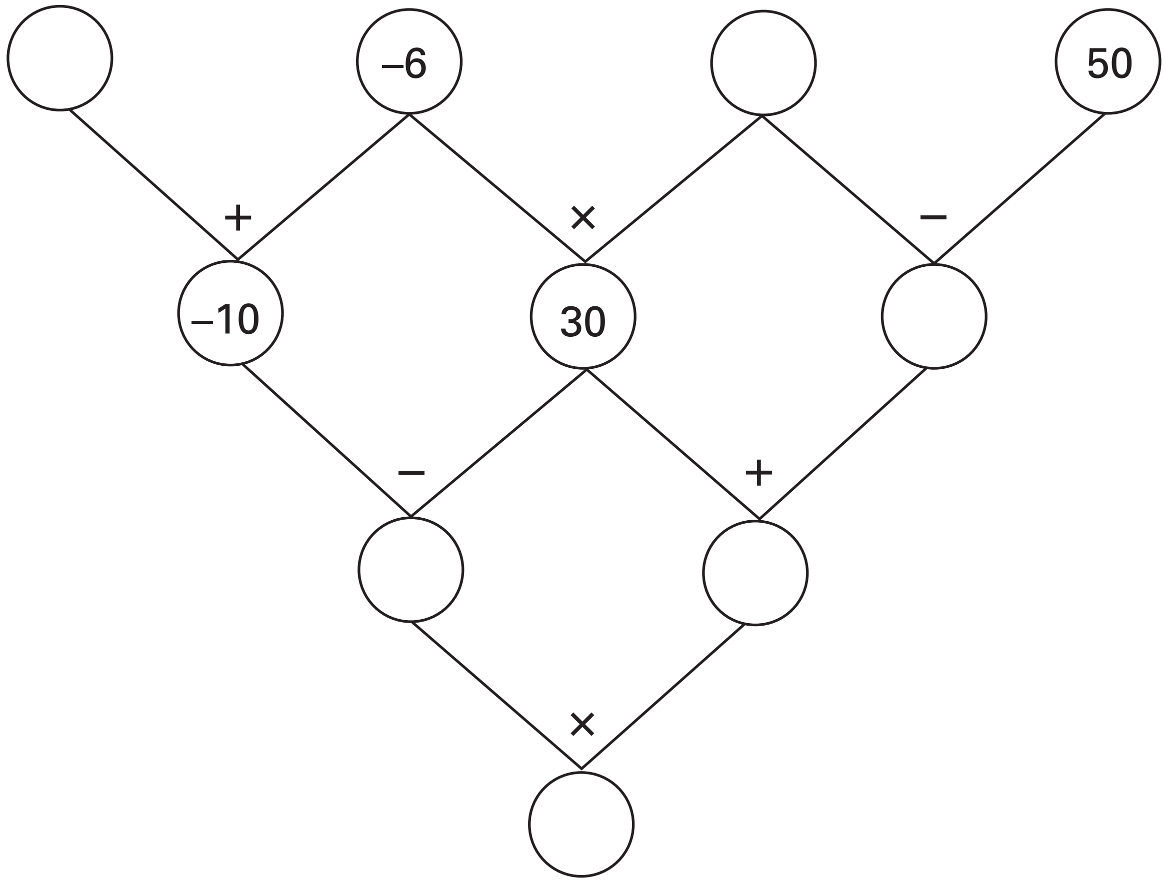
a. 2 × 12 _____ 3 × 12

b. −2 × 12 _____ −3 × 12

c. −5 × 9 _____ 5 × −9

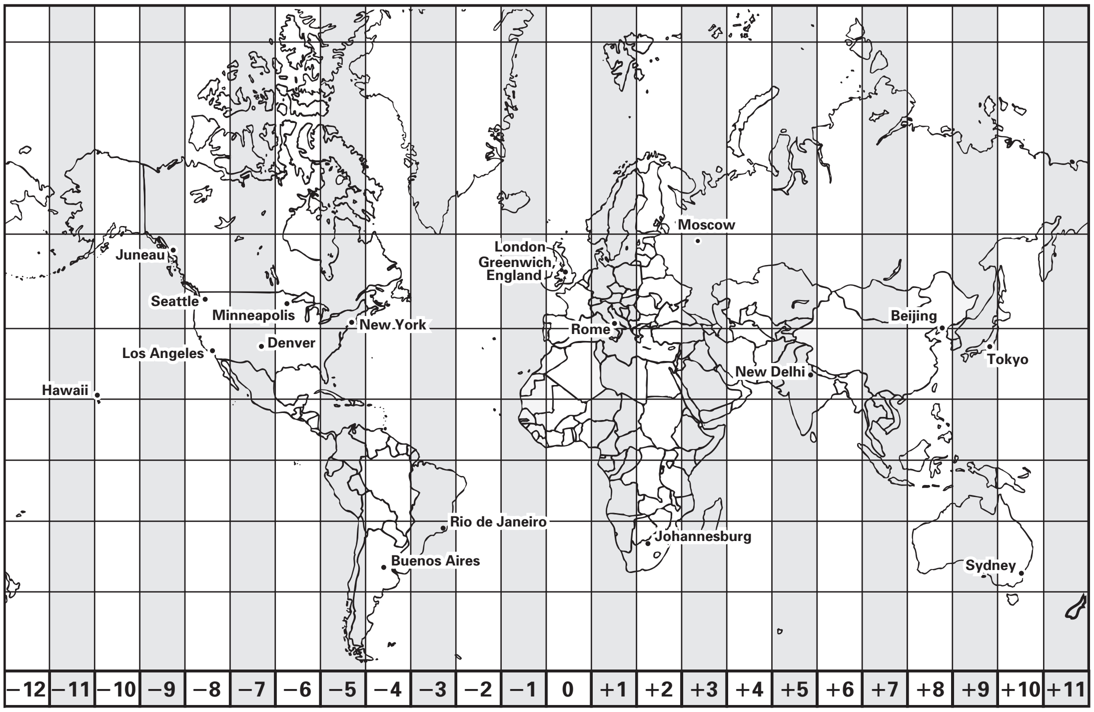
d. −5 × −9 _____ −5 × −8

4. Complete the operation tree.



Operations Unit Test

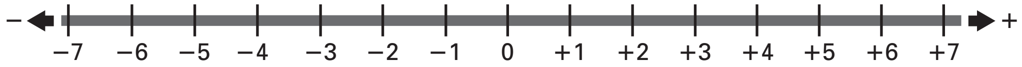
Use additional paper as needed.



1. a. Aisha, who lives in New York, wants to call her niece Nadia, who lives in Juneau, Alaska. It is now 8:00 in the evening in New York. What is the time in Juneau.?

b. Nathan traveled from Moscow (time zone marked +3) to Rio de Janeiro (time zone marked −3). How will he have to change his watch?

You may use the number line below to answer question 2.



2. Make true statements using <, =, or >.

a. -4 ____ 2

b. -4 ____ -5

c. $-3\frac{1}{2}$ ____ -3

d. -2.8 ____ $-2\frac{4}{5}$

3. Find the answer to the following calculations. You may draw a number line if that is helpful.

a. $(25) + (-15) =$

b. $(-60) + (-15) =$

c. $(-3.8) + (2.2) =$

d. $(25) - (-15) =$

e. $(-80) + (20) =$

f. $(-2\frac{3}{4}) - (-2\frac{3}{4}) =$

4. Compute the following.

a. $(5) \times (-8) =$

b. $(-4) \times (7\frac{1}{2}) =$

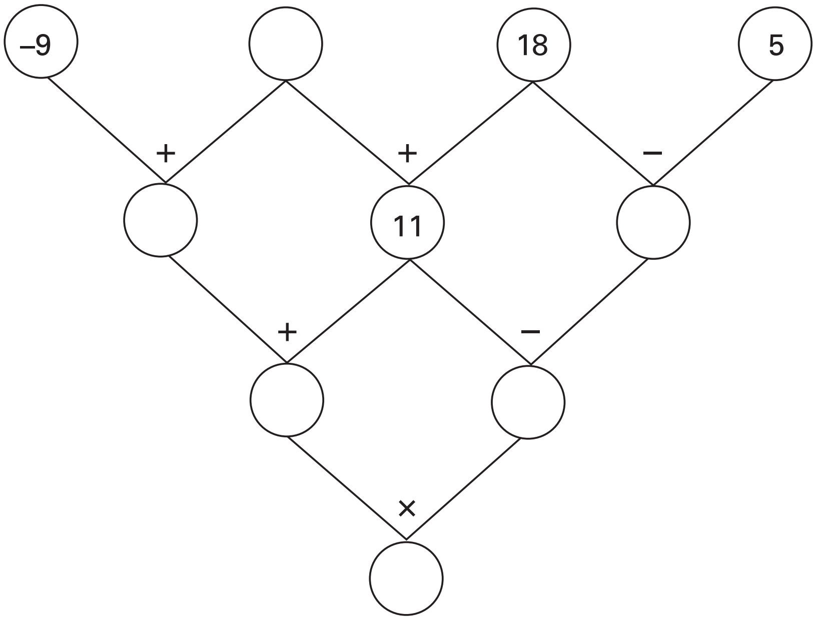
c. $(-10) \times (-12) =$

d. $(-20) \div (4) =$

e. What do you think the outcome of $(20) \div (-4)$ would be? Why?

Use additional paper as needed.

5. Fill in the missing numbers.

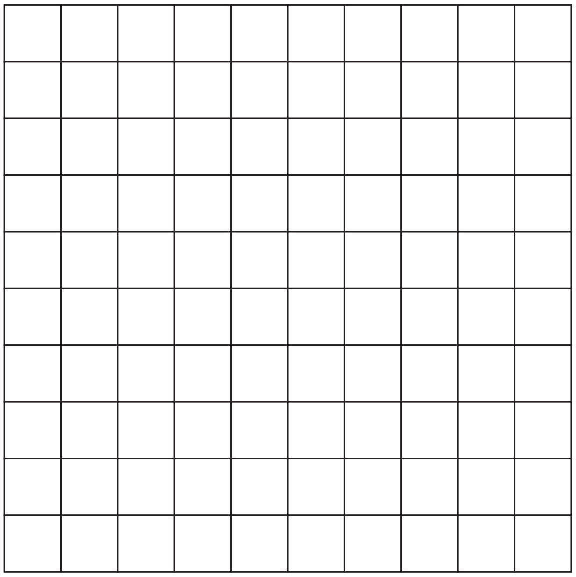


6. a. On the grid on the right, use a ruler to draw a coordinate system. Label the *x*-axis and the *y*-axis with a scale using numbers from −5 to 5.

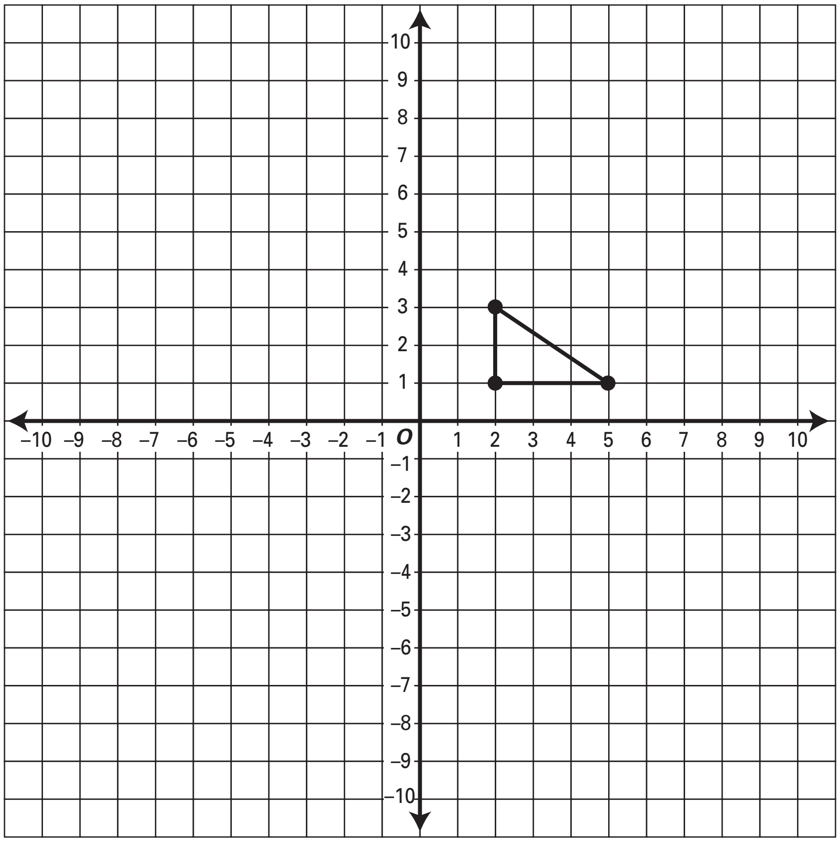
b. What are the coordinates of the origin?

c. In your coordinate system, plot the points $A\ (-2, -1)$, $B\ (3, -1)$, and $C\ (3, 2)$.

d. *A, B, C,* and an extra point *D* are the vertices of a rectangle. Write down the coordinates of *D.*



7. A right triangle has been drawn in a coordinate system.

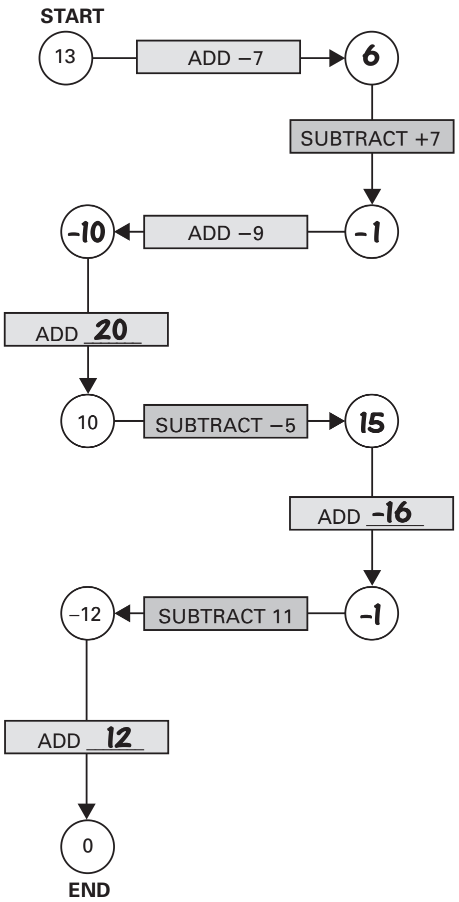


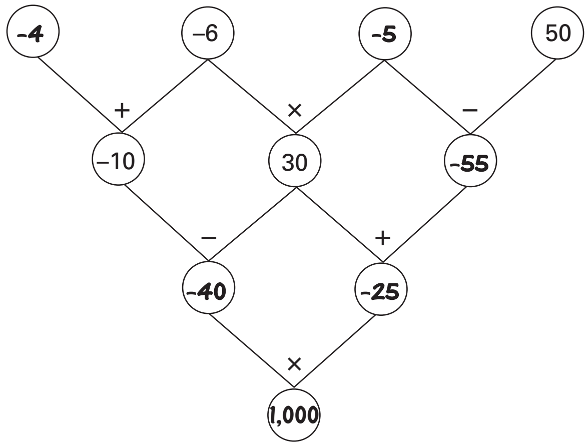
a. Add 2 to each of the coordinates of the triangle. What are the coordinates of the new figure? You may draw the new figure on the grid above.

b. Suppose you multiply all coordinates of the triangle *ABC,* as shown in the above figure, by −2. Describe what the new triangle looks like, compared to the original one. You may draw the new figure on the grid above.

c. By which number do you have to multiply the new triangle from part **b** to get the original triangle *ABC?* Support your answer with an explanation or a drawing.

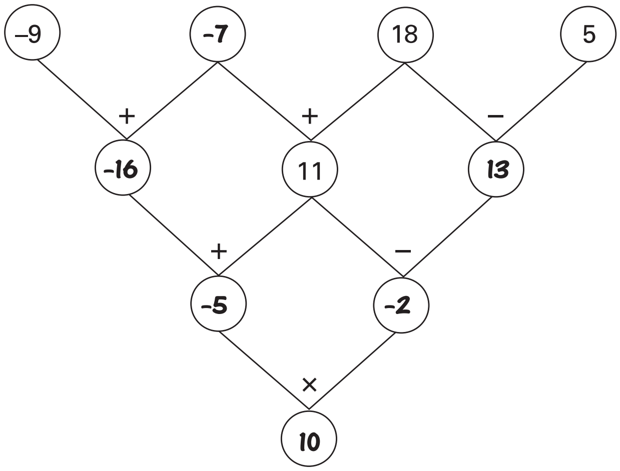
Operations Quiz 1
Solution and Scoring Guide

<table>
<tr><th>Possible student answer</th><th>Suggested number of score points</th><th>Problem level</th></tr>
<tr><td>1 a. Ernie spent money.
b. The amount he spent was $5
$24 \xrightarrow{+18} 42 \xrightarrow{-5} 37 \xrightarrow{-5} 32 \xrightarrow{+35} 67 \xrightarrow{+18} 85$</td><td>1
1</td><td>I/II
I/II</td></tr>
<tr><td>2. a. true
b. false
c. true
d. true
e. false</td><td>1
1
1
1
1</td><td>I</td></tr>
<tr><td>3. a. 130
b. 100
c. 25</td><td>1
1
1</td><td>I
I
I</td></tr>
<tr><td>4. The following numbers need to be filled in, in this order.

</td><td>4
(Follow students' operations and subtract 1 point for each wrong entry
Award credit if students give a correct result based on an incorrect answer to the previous operation.)</td><td>I/II</td></tr>
<tr><td>Total score points</td><td>14</td><td></td></tr>
</table>

Possible student answer	Suggested number of score points	Problem level
1. −13° F (or −13)	**2**	**I**
2. a. 31 **b.** −31 **c.** −100 **d.** −50	**1** **1** **1** **1**	**I** **I** **I** **I**
3. a. 2 × 12 < 3 × 12 **b.** −2 × 12 > −3 × 12 **c.** −5 × 9 = 5 × −9 **d.** −5 × −9 > −5 × −8	**1** **1** **1** **1**	**I** **I** **I** **I**
4. Numbers to be filled in are in bold: 	**2** **1** **2** **1**	**I** **I** **I** **I**
Total score points	**16**	

Operations Unit Test
Solution and Scoring Guide

Possible student answer	Suggested number of score points	Problem level
1. a. Four o’clock in the afternoon **b.** Nathan will have to set his watch six hours back because it is earlier in the day in Rio. (Award 1 score point if students just remark that the difference is 6 hours.)	**1** **2**	**I/II** **I/II**
2. a. $-4 < 2$ **b.** $-4 > -5$ **c.** $-3\frac{1}{2} < -3$ **d.** $-2.8 = -2\frac{4}{5}$	**4**	**I**
3. a. $25 + (-15) = 10$ **b.** $-60 + (-15) = -75$ **c.** $-3.8 + 2.2 = -1.6$ **d.** $25 - (-15) = 40$ **e.** $-80 + (20) = -60$ **f.** $-2\frac{3}{4} - (-2\frac{3}{4}) = 0$	**1** **1** **1** **1** **1** **1**	**I** **I** **I** **I** **I** **I/II**
4 a. $5 \times -8 = -40$ **b.** $-4 \times 7\frac{1}{2} = -30$ **c.** $-10 \times -12 = 120$ **d.** $-20 \div 4 = -5$ **e.** $20 \div -4 = -5$ Sample explanations: • $20 \div -4 = -5$ I reasoned that $20 \div 4 = 5$ because $4 \times 5 = 20$ and I know that $-4 \times -5 = 20$. • I reasoned $20 \div 4 = 5$ $-20 \div 4 = -5$ $20 \div -4 = -5$ $-20 \div -4 = 5$. • I think the rules are similar to those for multiplications.	**1** **1** **1** **1** **2** (Award 1 point for the correct answer and 1 point for a correct explanation.)	**I** **I** **I** **I** **III**

Possible student answer	Suggested number of score point	Problem level
5.  	**3** (Subtract one point for each wrong answer.)	**I**
6. a. Check whether axes are labeled correctly. **b.** (0, 0) **c.** Check whether the three points are plotted correctly. **d.** *D* (−2, 2)	**2** **1** **3** **1**	**I** **I** **I** **I**
7. a. *A* (4, 3); *B* (4, 5); *C* (7, 3) **b.** At least two of the three descriptions should be given. • The triangle is rotated (turned around 180 degrees). • The new triangle is shifted (moved over) to the left and down. • The new triangle is larger (exactly 4 × the area). **c.** Multiply by $-\frac{1}{2}$. Sample explanation: To get the original triangle you have to divide the new coordinates by −2. This is also the same as multiplying by $-\frac{1}{2}$.	**3** **2** **2** (Award 1 point for a correct answer and 1 point for a correct explanation.)	**I/II** **I/II** **II/III**
Total score points	**36**	

Glossary

The Glossary defines all vocabulary words indicated in this unit. It includes the mathematical terms that may be new to students, as well as words having to do with the contexts introduced in the unit. (Note: The Student Book has no Glossary. Instead, students are encouraged to construct their own definitions, based on their personal experiences with the unit activities.)

The definitions below are specific for the use of the terms in this unit. The page numbers given are from the Student Books.

coordinates (p. 46) the horizontal and vertical distances between a point and the origin; given by positive and/or negative numbers that represent the location of a point in a plane in relation to the origin

coordinate system (p. 46) a grid having intersecting horizontal and vertical number lines and in which a set of numbers is used to represent a point; divided in four quadrants

greater than (>) (p. 14) a phrase (symbol) used following a number whose value is greater than the value of the number following the phrase (symbol), for example, $5 > 2$

horizontal axis (p. 46) the number line going across the grid in the left/right direction

integer (p. 27) the set of positive and negative whole numbers, including zero

less than (<) (p. 14) a phrase (symbol) used following a number whose value is less than the value of the number following the phrase (symbol), for example, $2\frac{1}{3} < 2\frac{1}{2}$

mean (p. 28) a one-number summary of a set of data, sometimes called the average, with the arithmetic mean of a set of data resulting from adding the data and dividing the sum by the number of data values

number line (p. 12) a representation of numbers, ordered by size; a double number line has numbers on top as well as on the bottom

ordered pair (TG p. 46T) a set of two numbers that represents the location of a point in a plane, the first number being the horizontal distance from the origin and the second being the vertical distance from the origin

origin (p. 46) the fixed point of intersection of the horizontal and vertical axes

slope (TG p. 37T) the steepness or the slant of a line, determined by the ratio of the coordinates of two points on a line

time zone (p. 4) one of 24 geographical regions that divide earth, and within which the same standard time is used; approximately 15 degrees wide

transformations (TG p. 44A) the mapping, or movement, of all the points of a figure in a plane according to a common operation

vertical axis (p. 46) the number line going up and down on a grid

BRITANNICA

Mathematics in Context

Blackline Masters

Dear Family,

Very soon your child will begin the *Mathematics in Context* unit called *Operations*. Below is a letter to your child that describes the unit and its goals.

In this unit, your child will learn to add, subtract, multiply, and divide with positive and negative numbers. Students investigate time zones as an introduction to the addition and subtraction of positive and negative numbers.

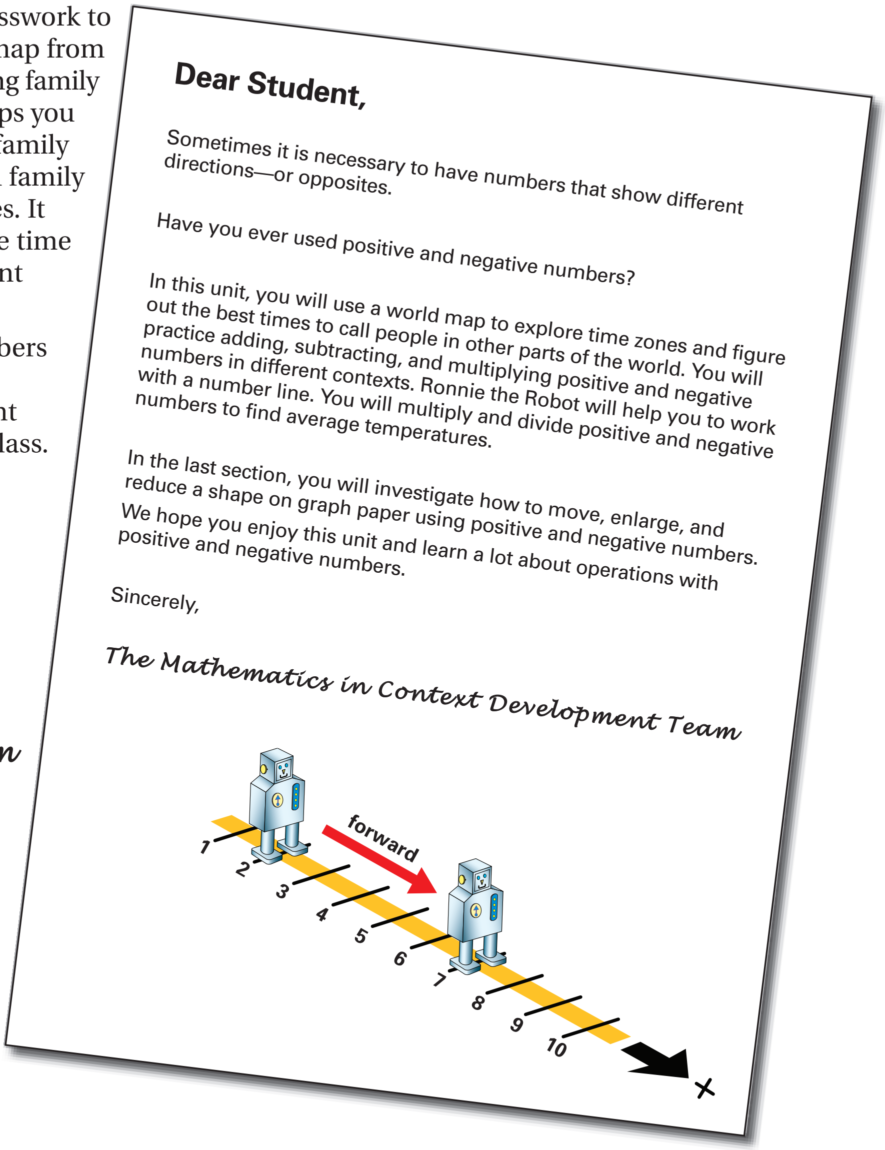
You can help your child relate the classwork to real life by investigating a time zone map from an atlas or encyclopedia and discussing family encounters with time changes. Perhaps you crossed one or more time zones on a family trip. Perhaps you telephone or e-mail family members who live in other time zones. It might be interesting to investigate the time stamp on email received from different time zones.

If you use positive and negative numbers in your work, share the ways you use them with your child. Your child might summarize this information for the class.

Enjoy helping your child learn about operations with positive and negative numbers.

Sincerely,

The Mathematics in Context Development Team



Dear Student,

Sometimes it is necessary to have numbers that show different directions—or opposites.

Have you ever used positive and negative numbers?

In this unit, you will use a world map to explore time zones and figure out the best times to call people in other parts of the world. You will practice adding, subtracting, and multiplying positive and negative numbers in different contexts. Ronnie the Robot will help you to work with a number line. You will multiply and divide positive and negative numbers to find average temperatures.

In the last section, you will investigate how to move, enlarge, and reduce a shape on graph paper using positive and negative numbers.

We hope you enjoy this unit and learn a lot about operations with positive and negative numbers.

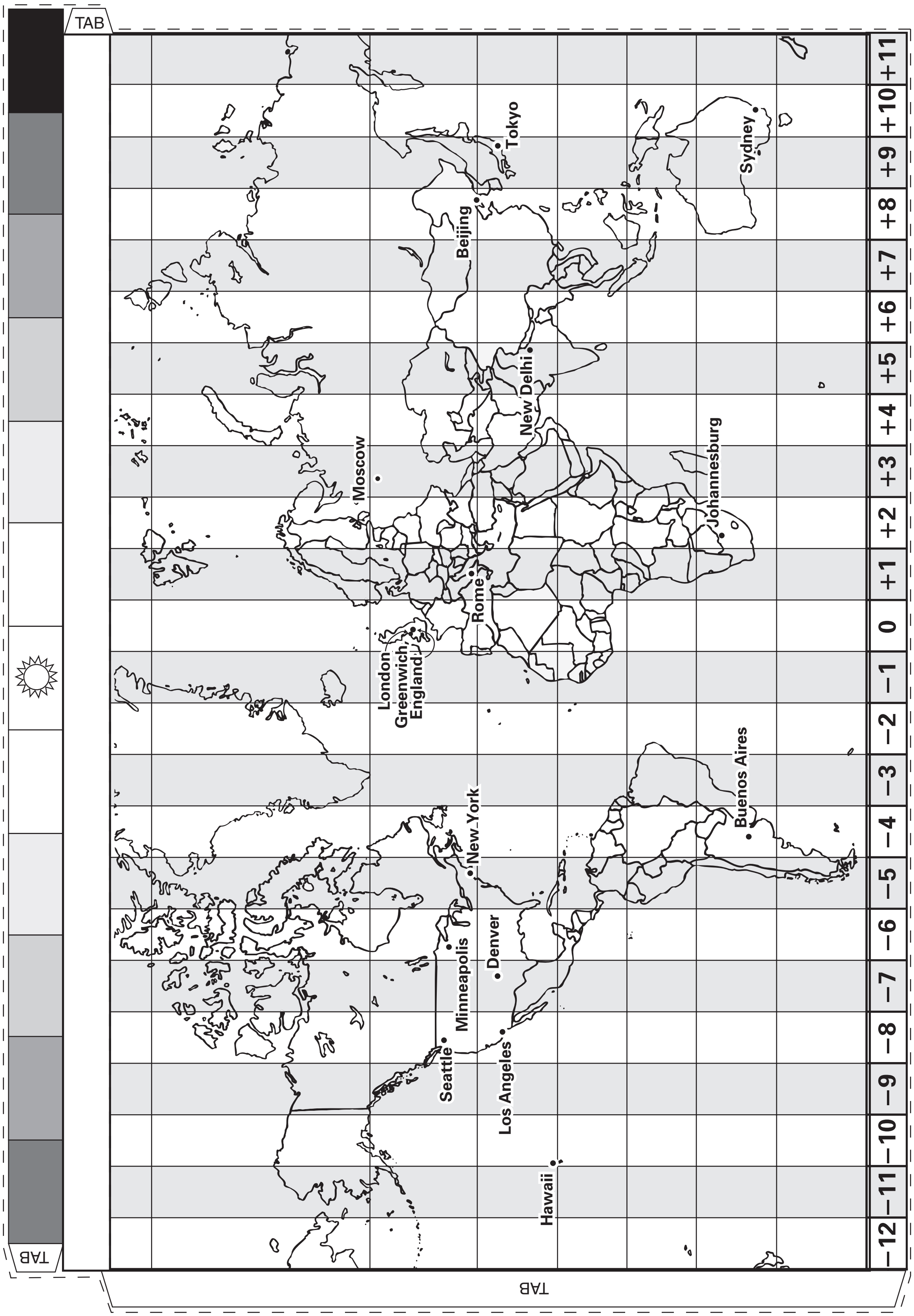
Sincerely,

The Mathematics in Context Development Team

Name ______________________________

Student Activity Sheet 1

Use with *Operations*, pages 3 and 5.

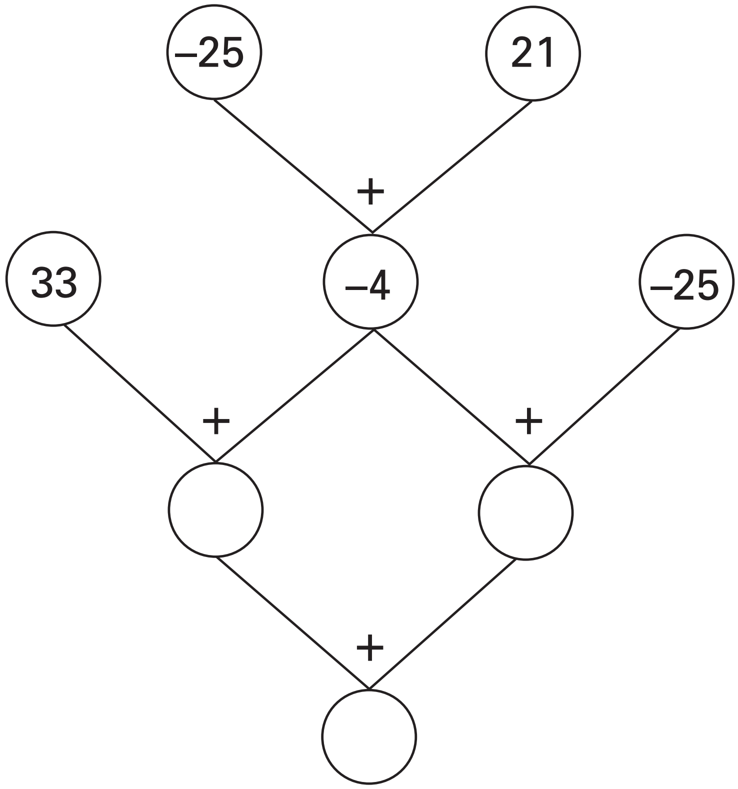


Student Activity Sheet 2

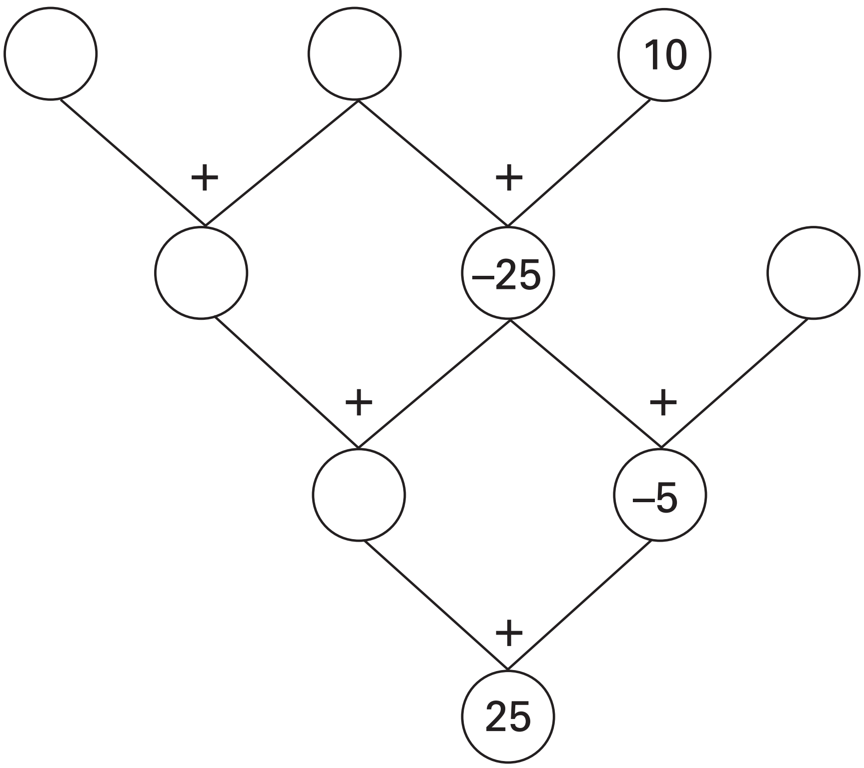
Use with *Operations*, page 25.

Name ______________________________

a.



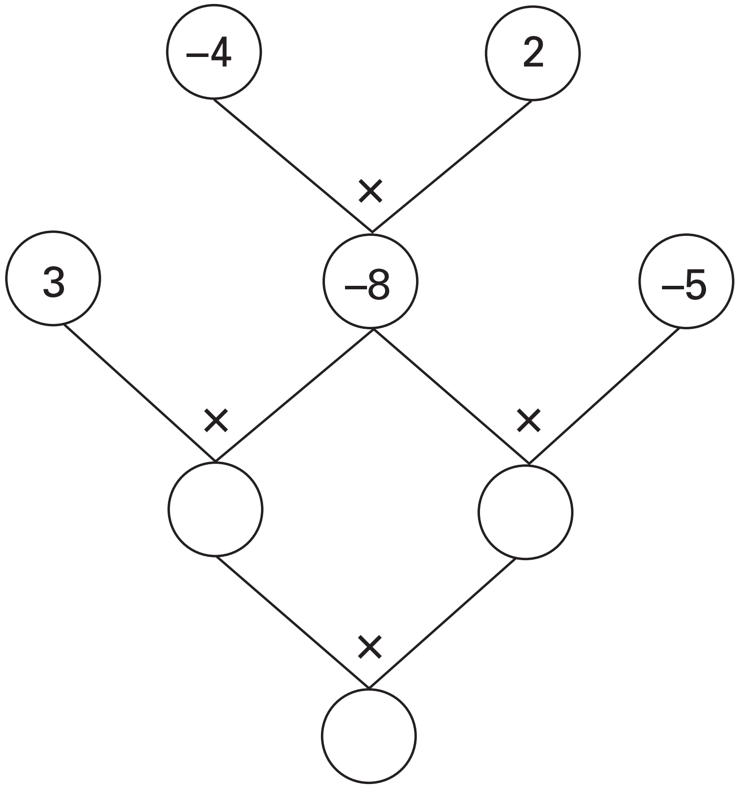
b.



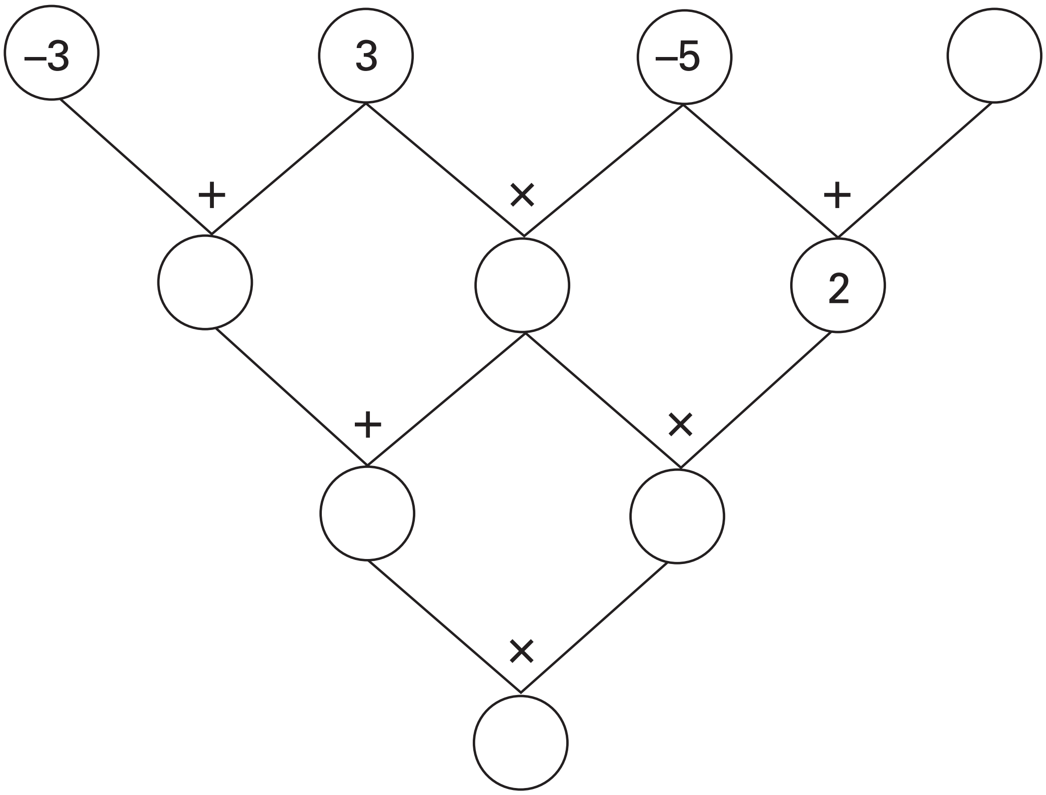
Name ________________________________

Student Activity Sheet 3
Use with *Operations*, page 41.

a.



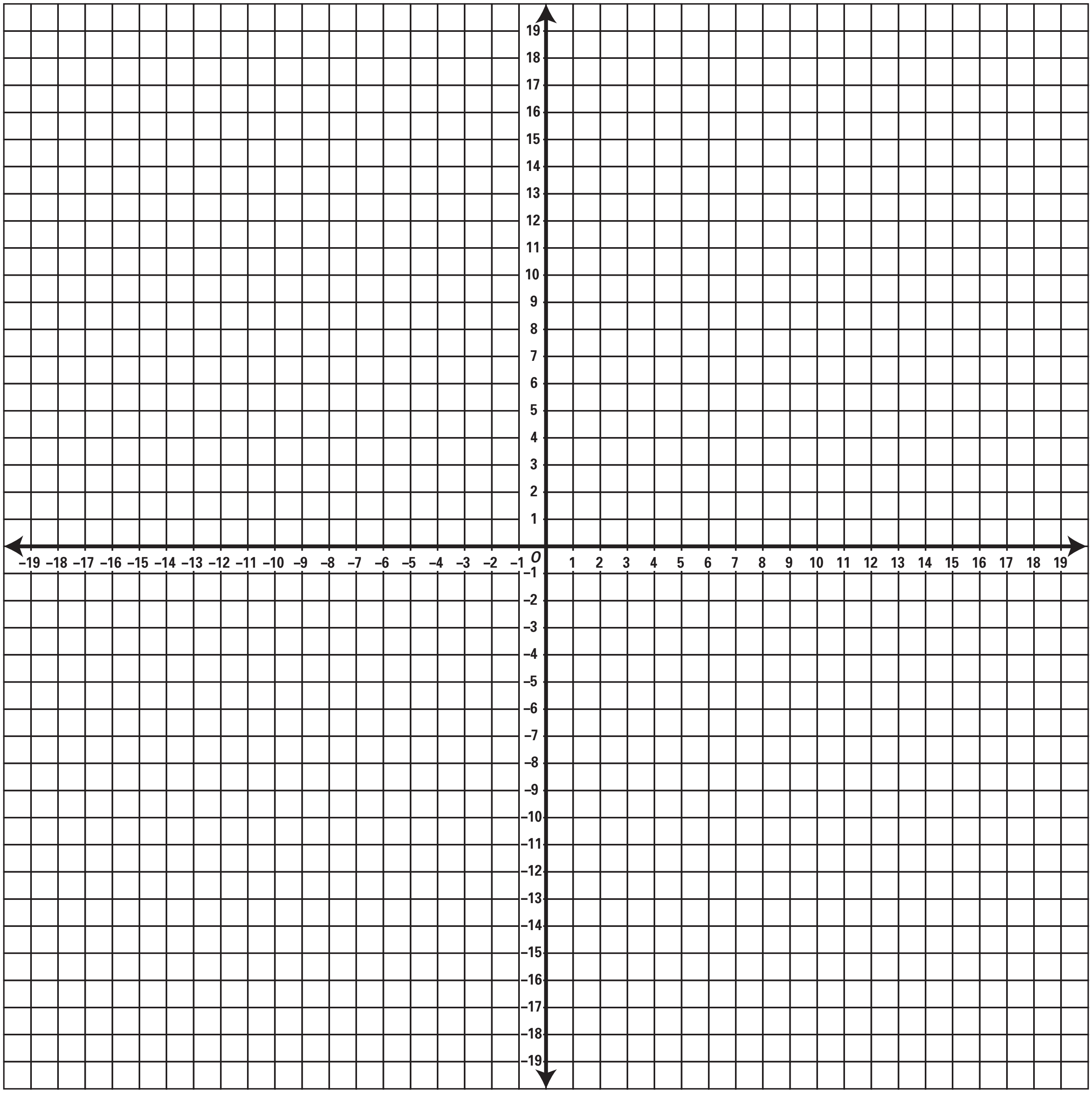
b.



Student Activity Sheet 4

Use with *Operations*, pages 47–49.

Name ______________________________



Name ____________________________________

Student Activity Sheet 5

Use with *Operations*, page 57.